

# UNITED STATES NAVAL POSTGRADUATE SCHOOL



NOTES ON THE APPLICATION OF VARIOUS  
THEORIES AND TECHNIQUES FROM STRENGTH  
OF MATERIALS WITH RESPECT TO SHIP  
DESIGN AND CONSTRUCTION

by

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The notes that follow attempt to show a practical application to ship design and construction of the topics covered in the ME 521 and ME 522 course sequence. A large number of references have been included simply to acquaint the reader with some of the more readily available publications. A lot of material has been covered. The inadequacy of discussion will be obvious, but hopefully may arouse some curiosity.

A large amount of time will be allotted to beam theory in the courses mentioned. Take this as a point of departure and see where the beam theory, as applied to ship structure, will lead.

Ships are hollow structures that carry different things to different places for different reasons. The external shape, as defined by the moulded lines, is determined by factors such as purpose of ship, desired capacity and powering, with no regard to strength.<sup>1</sup>

The resulting hull, when structurally assembled, can be looked at as a flexible hollow box girder (beam) supported by a varying elastic support, the water. Figure 1 describes the shape of a typical merchant ship hull. The three views noted are actually represented to scale on a "lines drawing", an example of which is shown in Figure 2.

1 Numbers in superscript position refer to references listed at end of notes.



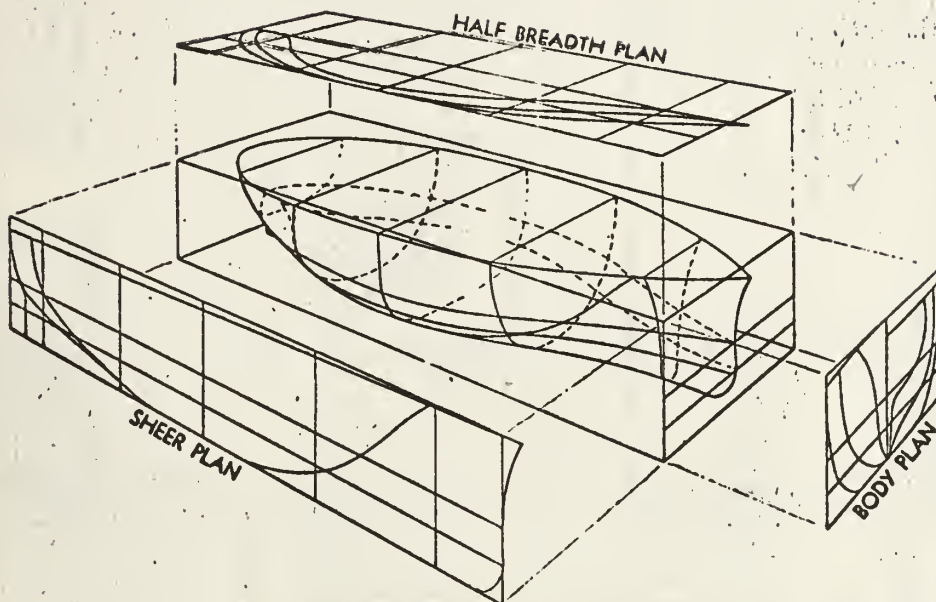


Figure 1. Projection of Ships Lines.

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# Principal Dimensions

Length over all	255' 4"
Length on designer's LWL	249' 0"
Length between perpendiculars (Classification Society)	243' 0"
Breadth, molded	40' 0"
Depth, molded at 100, Upper Deck	18' 9"
Draft, molded, DLWL	16' 0"
Displacement, molded, salt water	3180 tons
Deadweight, (approximate)	2000 tons

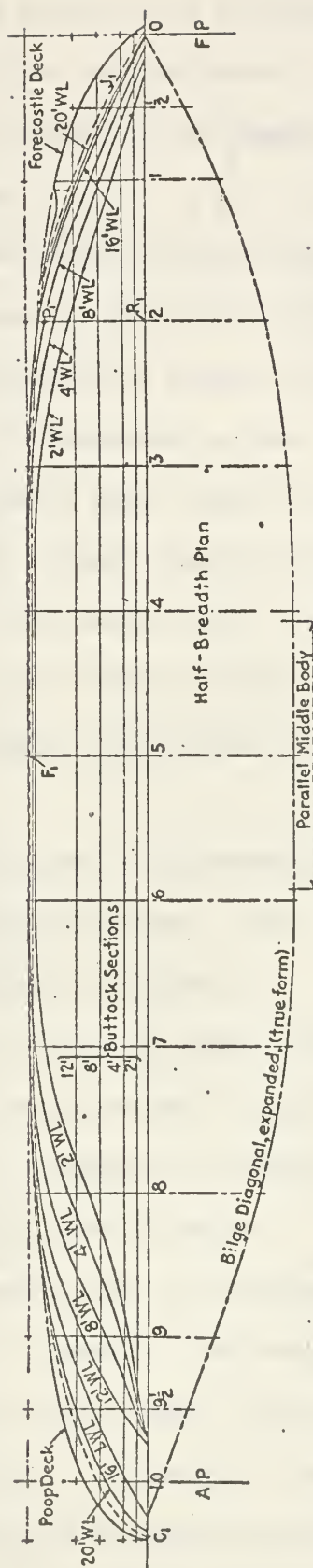
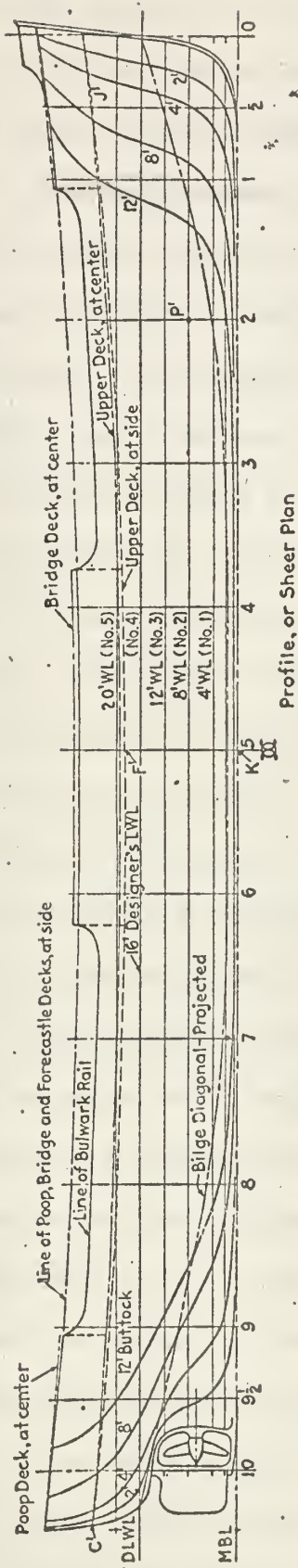
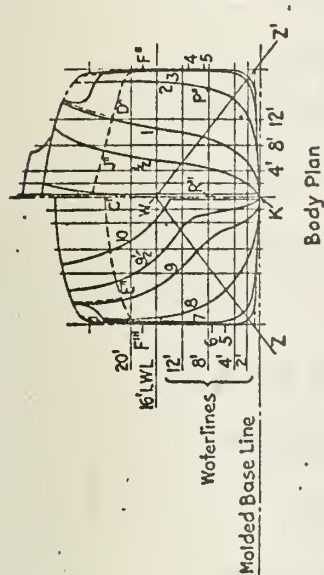


FIG. 2.—LINES DRAWING OF CARGO STEAMER

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From this drawing the "curves of form" can be constructed by plane and volume integration of the moulded lines.\* A sample of these curves is shown in Figure 3. Of immediate interest is the displacement curve.

For the moment, consider a rectangular barge floating in still water, Figure 4, half-loaded as indicated by the sectioned area. The loading curve (downward forces, including the weight of hull structure) can be superposed on the buoyancy curve (the volume of displaced water times the density of the water), which is the support (upward forces). The result of the superposition is the shear curve. As usual, the integral of the shear curve is the bending moment. It is the bending moment which the barge structure must resist at that particular loading.<sup>2</sup>

A different loading will bring about a different weight curve and ultimately a different bending moment. Note buoyancy curve and variation with draft in Figure 3.

It is this procedure which is applied in actual ship design to determine hull structure requirements - that is, the development of midship section, analogous to determination of the section modulus of a beam ( $Z$ ), the  $I/c$  ratio.

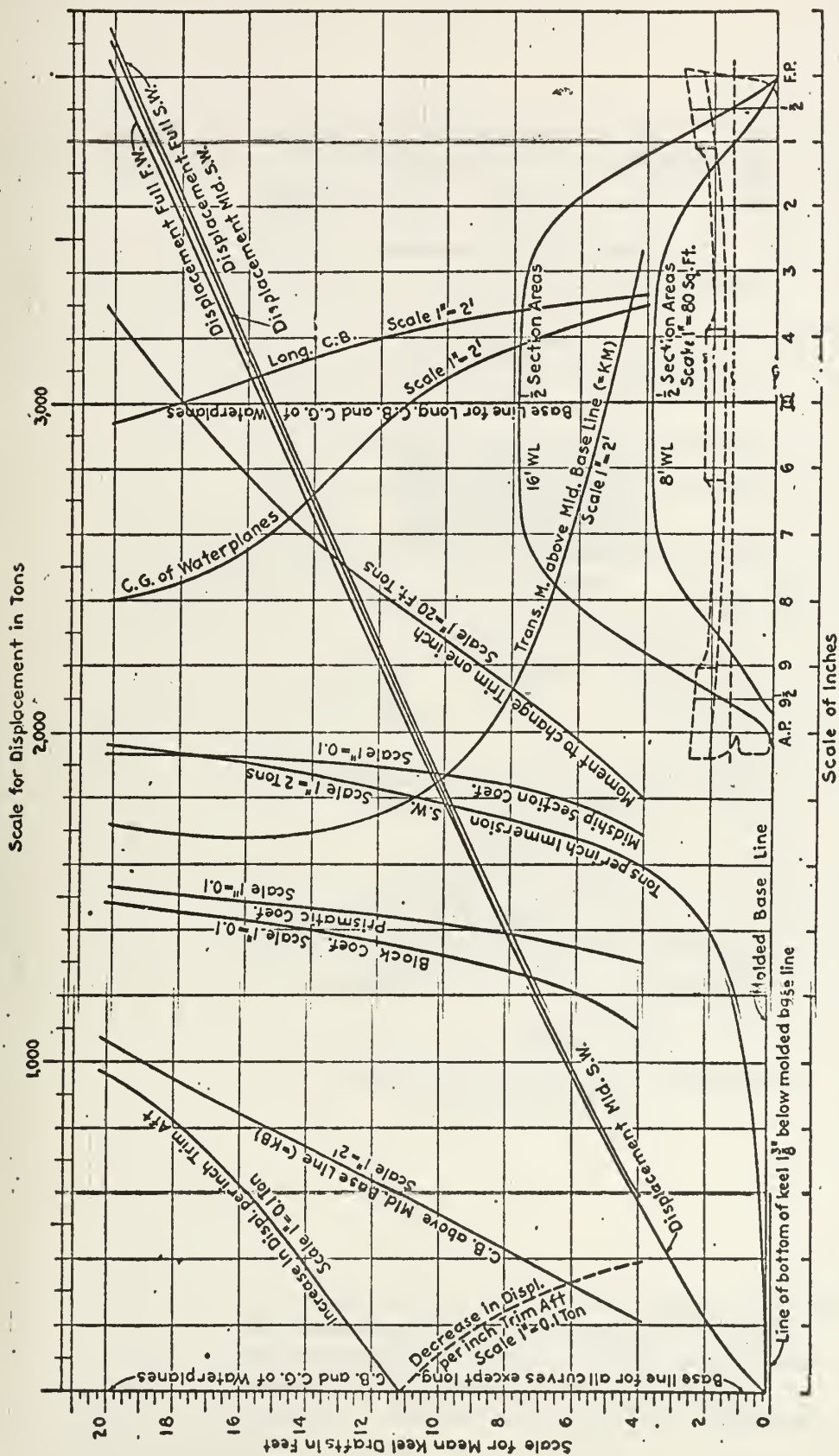
Conditions other than still water must be investigated. A standard wave profile is shown in Figure 5. The standard wave is based on the trochoid, its length equal to that of the ship and the wave height is  $1/20$  of the length. The U.S. Navy uses a slightly modified form in warship design.<sup>3</sup>

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\* Generally by Simpson's Rule. U.S. practice normally uses the trapezoidal rule. These numerical methods and computer application will be covered in MA 416. Numerous papers have covered the subject, including the fairing of ships lines by digital computer methods.









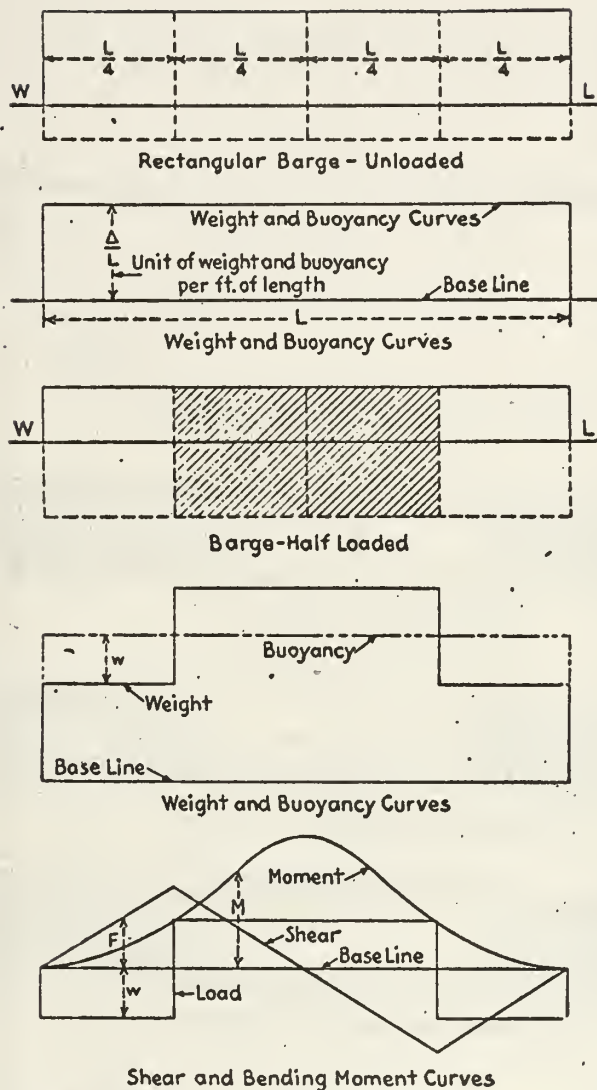


FIG. 4.—BENDING MOMENT DEVELOPMENT OF RECTANGULAR BARGE IN STILL WATER

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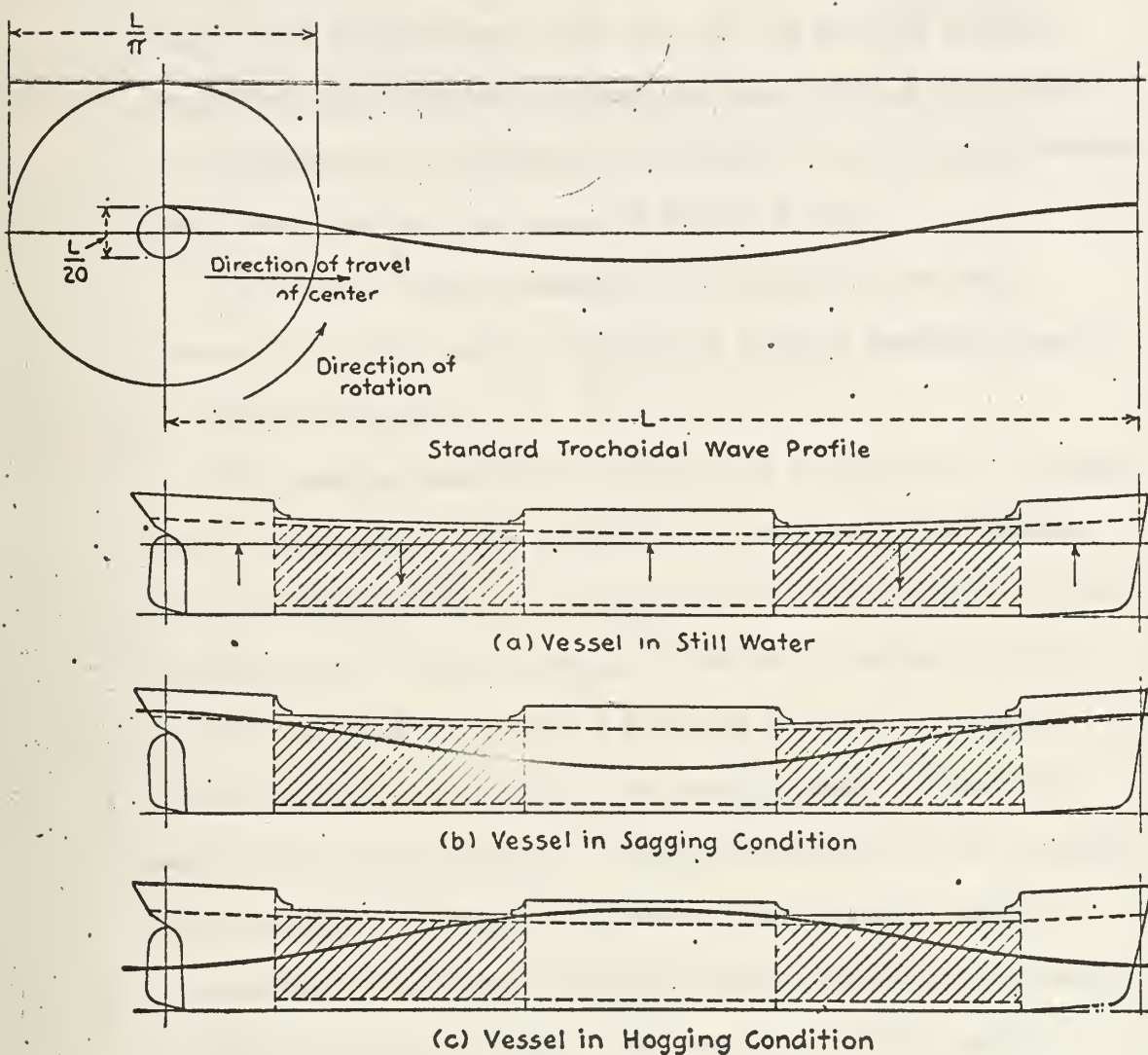


Figure 5. Superposition of Wave Profiles.

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The imposition of a wave pattern modifies the buoyancy curve. The hogging and sagging conditions which produce maximum tension and compression in the area of the maximum bending moment are examined for resulting maximum stress. The location of the maximum bending moment is usually at the midships section. These wave profiles are shown in Figure 5 also.

The simple barge procedure, now applied to a ship, produces the still water, hogging and sagging bending moments as shown in Figure 6.

The bending moment that must be met is now known, and the shape of the beam at the midpoint (as defined by the midship section moulded lines in the body plan) is also known. It is now necessary to develop the distribution of effective hull material to give the required  $Z$  of the section, or midship section modulus, to satisfy the bending moment within the safe limit of the material. Effective material of the section, for bending, is variously defined as that material which is longitudinally continuous throughout the midships half-length or three-fifths half-length. The resulting midship section might look something like that pictured in Figure 7.<sup>4,5,6,7,8</sup>

A slightly different configuration is shown in Figure 8, and a representative destroyer section in Figure 9. More will be said of naval construction later. Up to this point there is no great difference between naval and merchant preliminary design practice.

Bulkheads, frames, superstructure and so forth contribute little, if any, to longitudinal strength. Superstructure is considered non-effective material and is designed to be such, which is one reason why expansion joints are to be found.









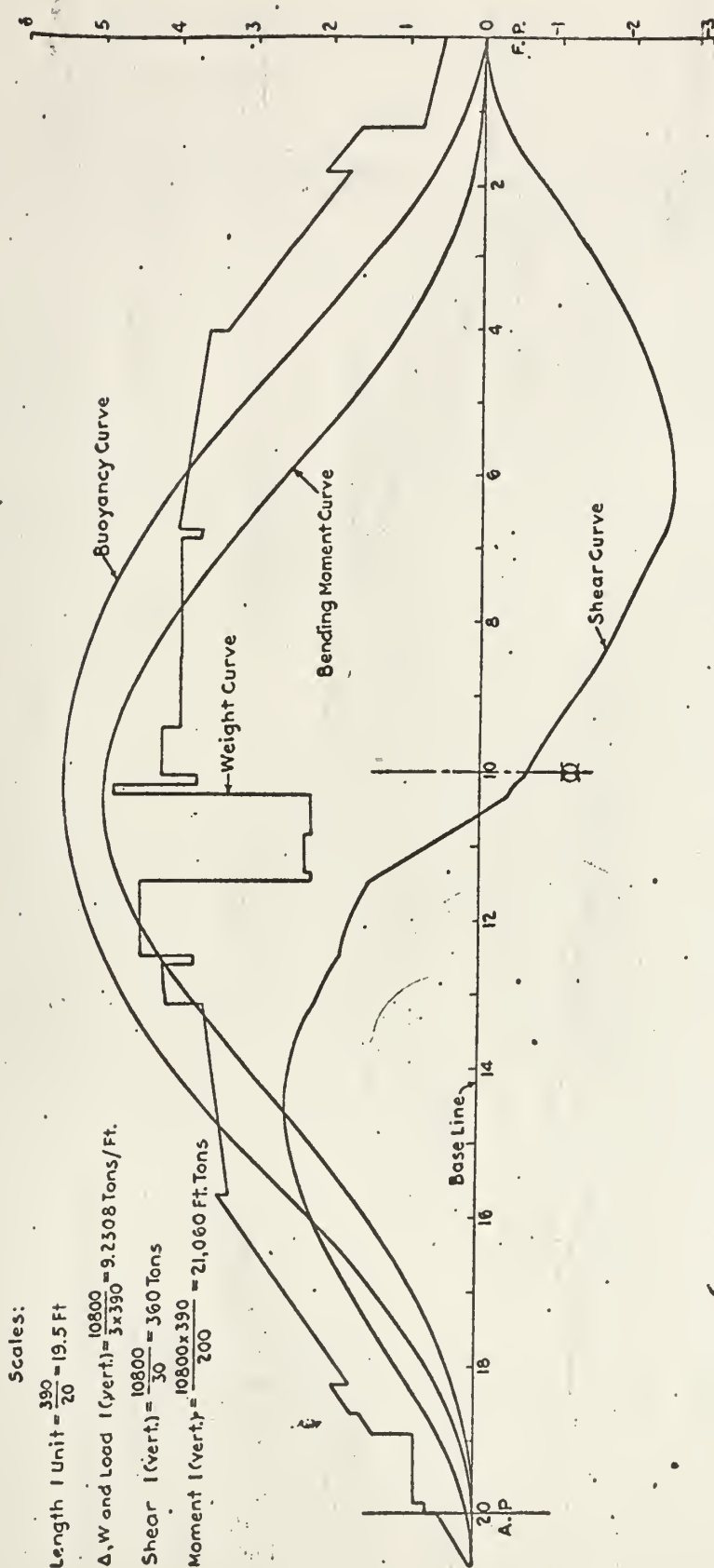


FIG. 6 -STRENGTH CURVES. HOGGING CONDITION. TYPICAL FREIGHTER, 390 FEET BY 55 FEET BY 30 FEET 6 INCHES

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Scales:

$$\text{Length 1 Unit} = \frac{390}{20} = 19.5 \text{ Ft.}$$

$$\Delta, W \text{ and Load 1 (vert.)} = \frac{10800}{3 \times 390} = 9.2308 \text{ Tons/Ft.}$$

$$\text{Shear 1 (vert.)} = \frac{10800}{30} = 360 \text{ Tons}$$

$$\text{Moment 1 (vert.)} = \frac{10800 \times 390}{200} = 21,060 \text{ Ft. Tons}$$

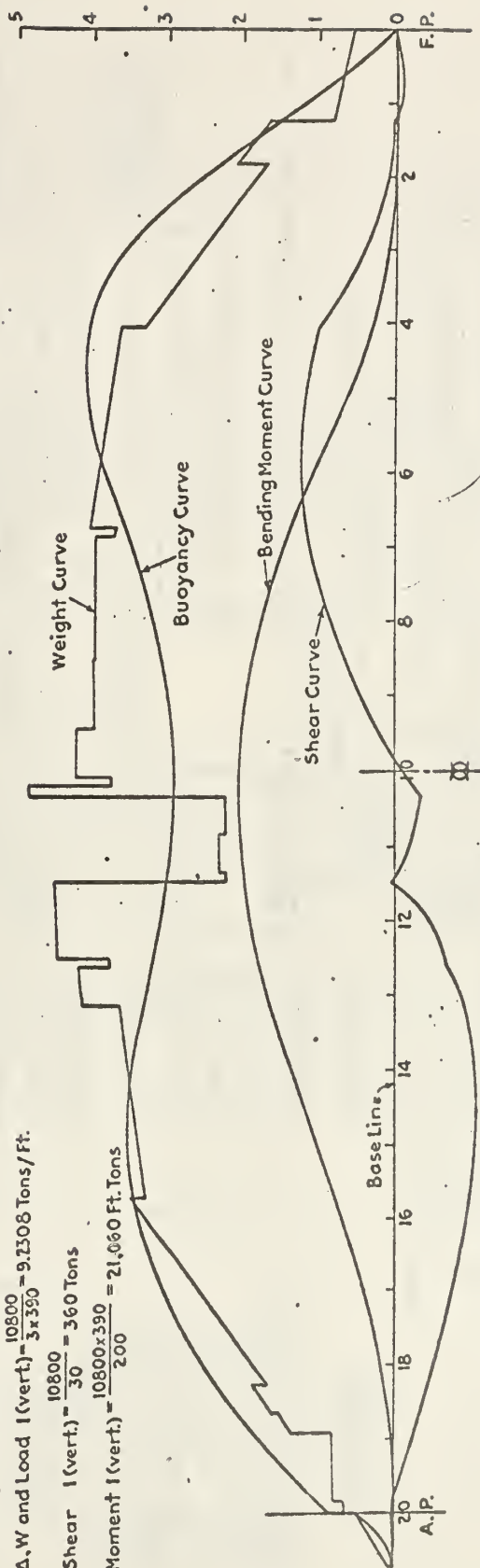


FIG. 6.—STRENGTH CURVES. SAGGING CONDITION. TYPICAL FREIGHTER, 390 FEET BY 55 FEET BY 30 FEET 6 INCHES

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Deck Plating 48 lb to 55 lb of Ends  
Hatch Strokes 37 lb  
Deck Area Based on One 20" Cut  
in Dk on Each Side

Upper Deck Long's D.A. 8'x4'x5/8"

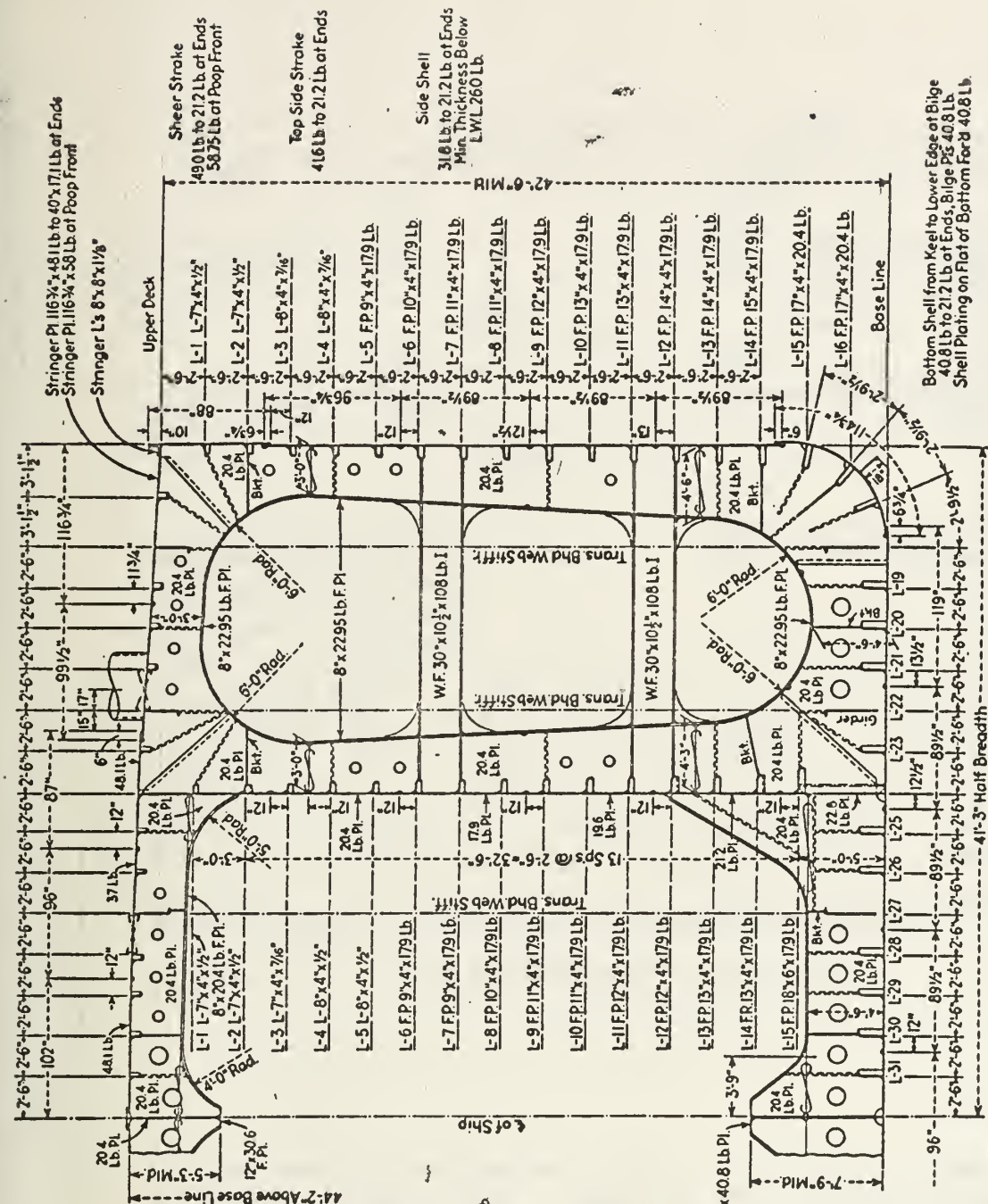


FIG. 7 MIDSHIP SECTION, 26,800-T-DWT TANKER, PLAIN BULKHEADS

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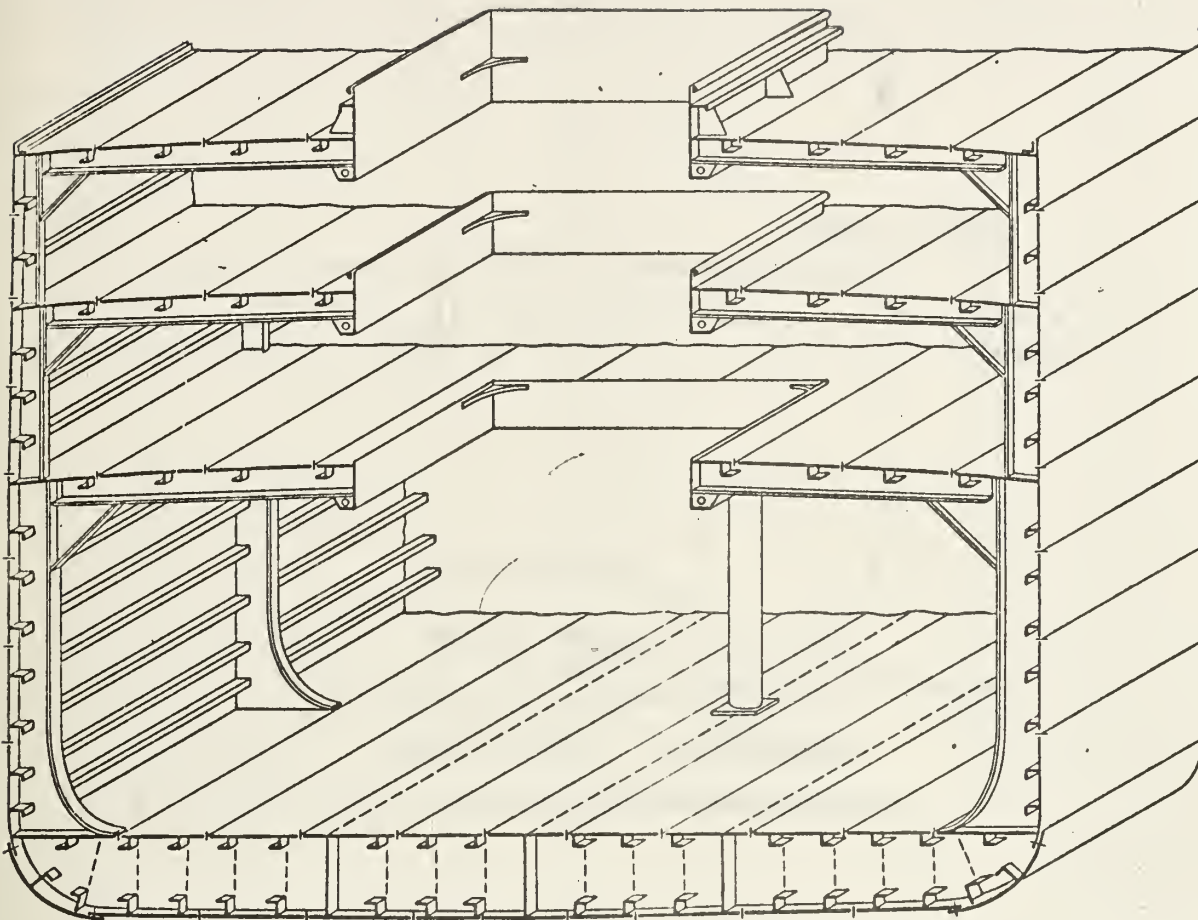


FIG. 8 LONGITUDINALLY FRAMED MIDSHIP SECTION

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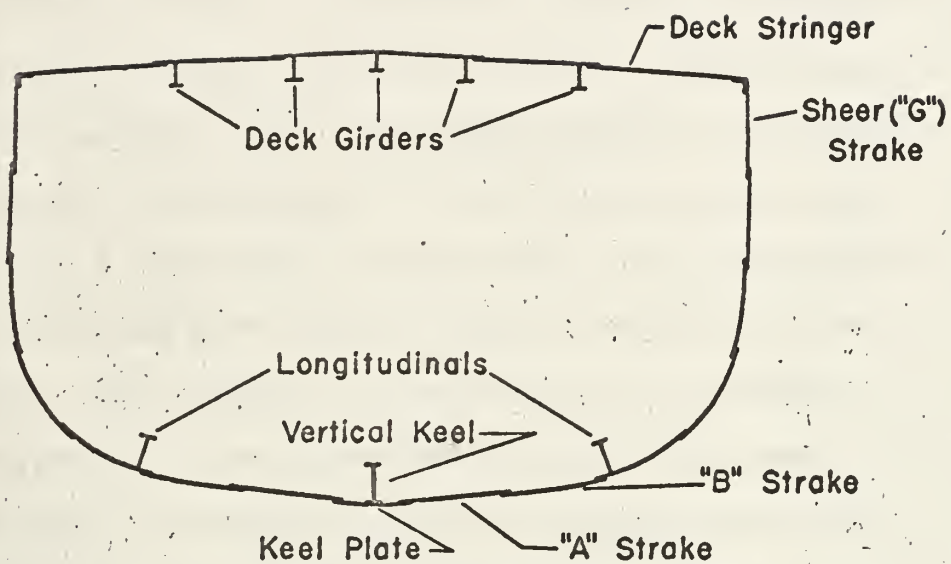


FIG. 9 Destroyer Longitudinal Strength Members.

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The structure has been developed based on an assumed still water bending moments. This does not mean that the resulting stress condition is the maximum that the hull might actually encounter, nor does it mean that the bending moments are the only loading applied to the hull girder.

A ship, when moving through waves, experiences torsional (wracking) stresses. Although of a relative minor importance, torsional strength of ships has been the subject of numerous investigations.<sup>9,10,11</sup> The maximum condition is encountered when the wave profile is such that there is a high on one bow and a high on the opposite quarter. This condition is usually found when running at about 45 degrees to the wave train. This situation further complicates the buoyancy curve and it is most convenient to look at "odd" wave positions as contributing increments or decrements to the standard pattern of still water loading in terms of pounds (or tons) per unit length. The resulting stresses can be quite high, and have contributed to ship girder failures.

Even in still water there is another loading which has been neglected. That is the hydrostatic load on the hull due simply to immersion in the water. This is a relatively simple problem and the usual method of attack is to treat it as a normal stiffened plate (stiffening is provided by transverse frames and longitudinals).

The principal hull structure resisting torsion are the bulkheads, deep web frames and the double bottom structure.

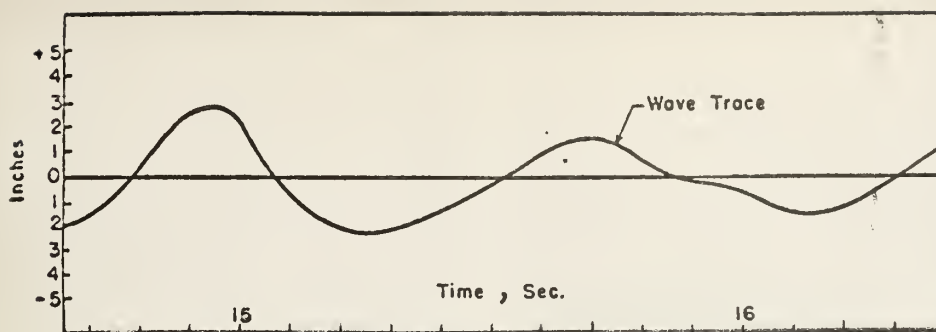


Ship slamming, a term used to denote the forcible re-entry of the bow into the water, greatly increases the stress and causes marked perturbations in the normal time varying bending moment,<sup>12,13,14,15,16</sup> which is a function of ship speed and wave length. A compound picture of this phenomenon is shown in Figure 10. Accompanying the slamming, really as a product of it, are the familiar shudders and shakes that all have experienced when trying to make knots in heavy weather. Slamming has been the subject of rather intense investigation during the past years.<sup>17,18,19</sup> Although an analytical statement of ship motion in an arbitrary seaway still cannot be completely made, many have tried, and with considerable success.<sup>20</sup>

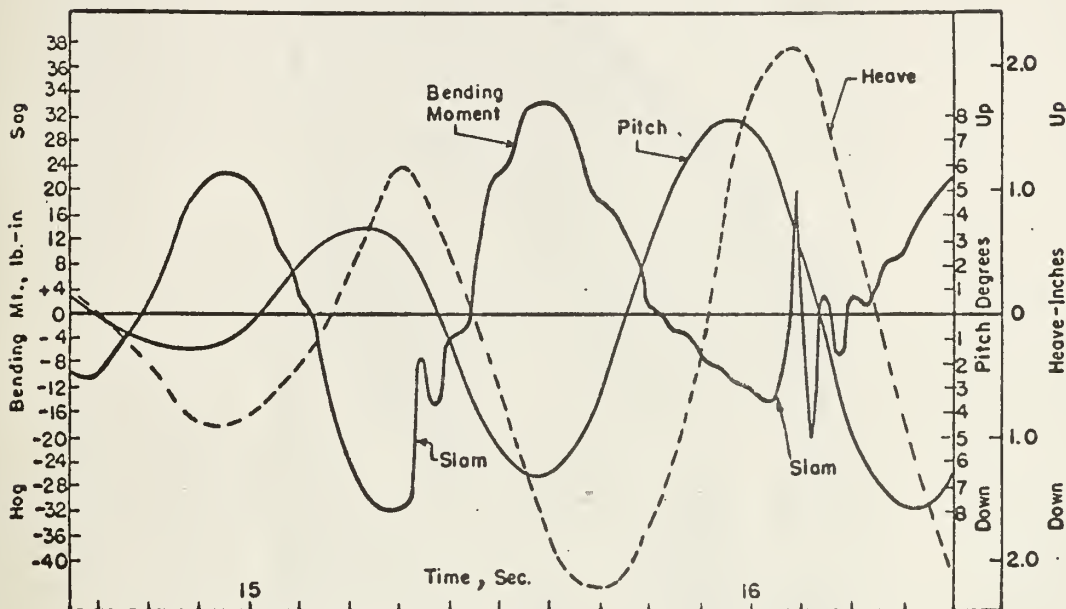
A ship's hull is, in fact, full of holes. Hatches, ports, discharge and inlet openings and so on are profuse. This, imposed on a complex beam, subject <sup>to</sup> ~~at~~ complex loading, does not simplify the design problem. Stress concentration is a very important consideration. As the ship progresses through the waves the bending moment will range from a maximum in tension to a maximum in compression, and as a result fatigue enters the picture, not only in bending, but in torsion also.

Prior to examining the possible effect of these complicating features, the question may be asked as to just how good the basic beam theory is in its application to a ship hull, the theory on which the midship section and essentially the whole hull was determined. Picture again the basic structure. Figure 11 depicts a structural scale model before shell plating and decks have been added.

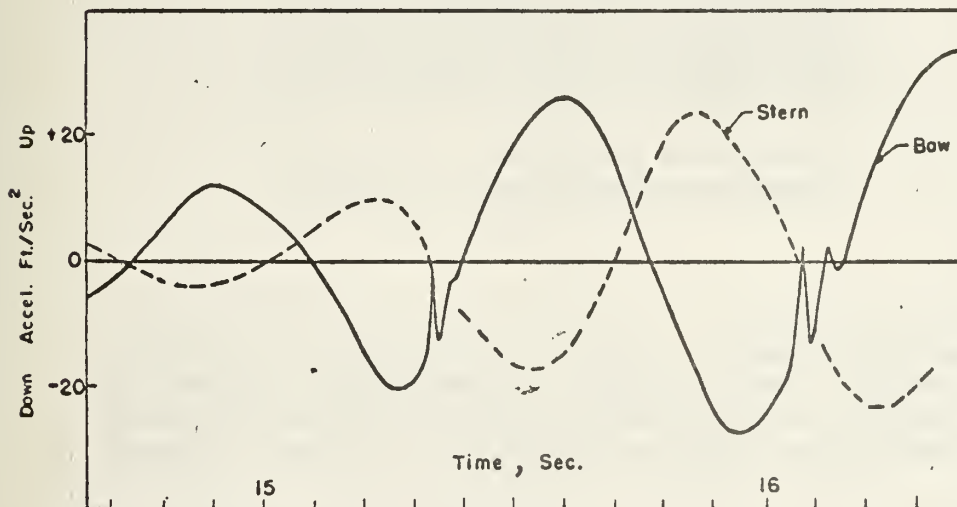




WAVE TRACE AT POINT OF CARRIAGE  
ABREAST MODEL FORWARD PERPENDICULAR



HEAVE, PITCH AND BENDING MOMENT



VERTICAL ACCELERATIONS AT BOW AND STERN

Figure 10.

MOTIONS, ACCELERATIONS, AND BENDING MOMENTS IN IRREGULAR HEAD SEAS AT SPEED CORRESPONDING TO 14½ KNOTS, SHOWING MAXIMUM BENDING MOMENT AND TYPICAL SLAMS; MODEL OF T2-SE-A1 TANKER

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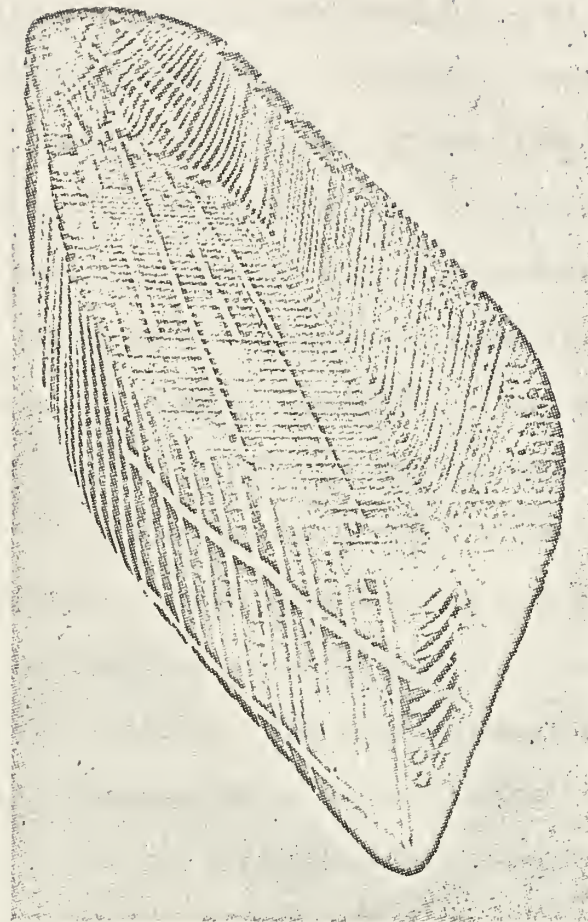


Figure 11. Structural Scale Model,  
Plating and Decks not yet Attached.

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The proof is in the pudding and the answer was, in fact, found in full scale tests, some to failure of the hull girder. Some fifteen separate ships have been tested. In the United States the first noteworthy tests took place in 1930.<sup>21</sup> Riveted, welded and combination construction was represented in the ships tested. The majority of the tests took place during World War II and in the years immediately following. Some tests were conducted for a particular reason which will be examined later. Ship structure research has not ceased.<sup>56</sup>

The tests did, however, substantiate the use of the simple beam approach. Failure in ships of mild steel construction took place at an average stress of about 27,000 psi, the failures were in compression. Both upper deck and keel failures were obtained in both hogging and sagging bending moments, but all were compressive failures. The specific conclusion reached was: the hull, when subjected to longitudinal bending moments, behaves as a hollow box girder (beam) in bending in accordance with elementary theory. Figures 12 a, b, and c picture the actual observed stresses in test of a destroyer, tanker and dry cargo ship respectively.<sup>21</sup>

The full scale tests also confirmed and laid the foundations for laboratory tests and searching analytical treatments. Usually, these investigations have been limited to small sections of stiffened structure termed "grillages". A typical grillage is shown in Figure 13. Commodore H. A. Schade, USN(ret) of the University of California, Berkeley, is an authority on this subject and has published extensively.



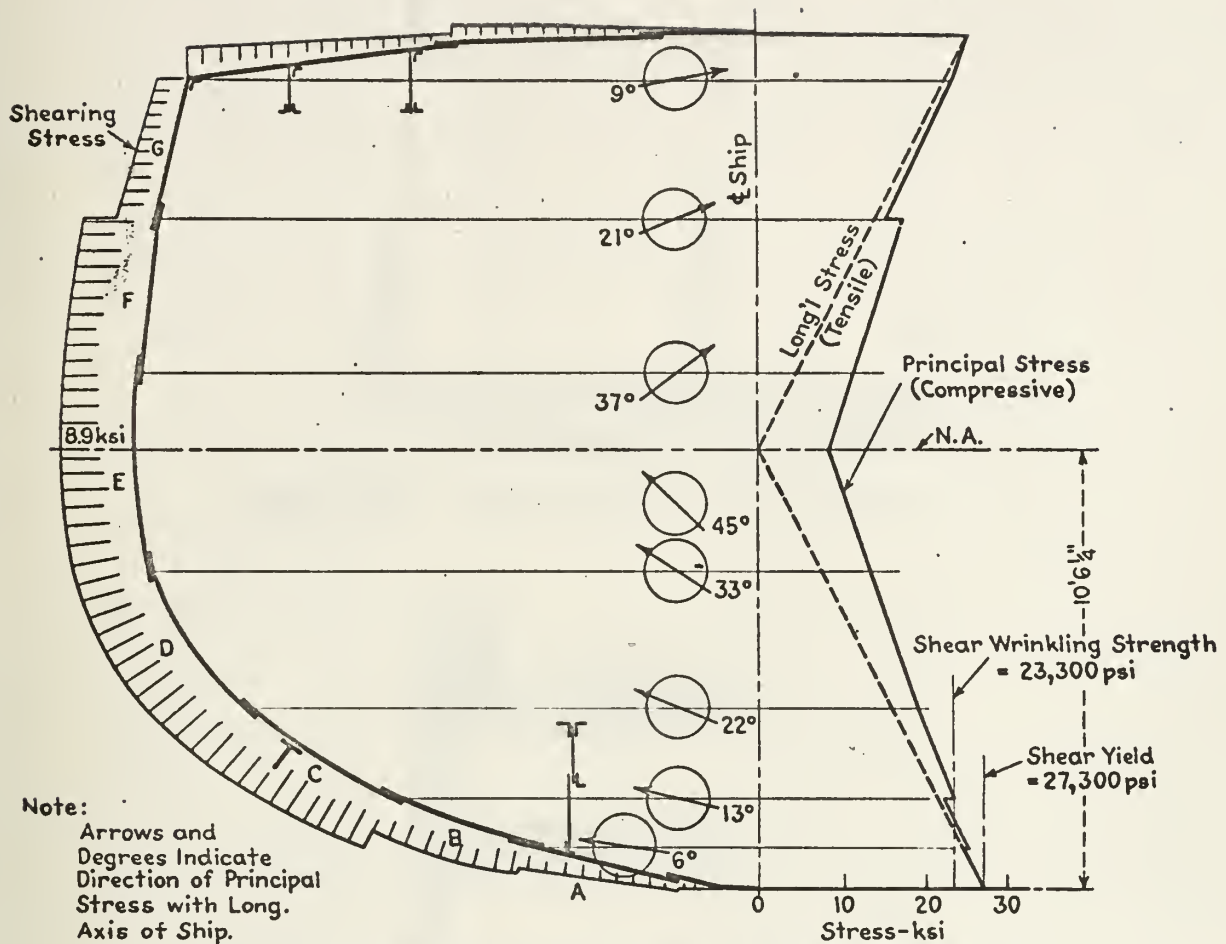


Fig 12a. Principal stress analysis—USS Bruce. Stress at failure—Frame 102 1/2

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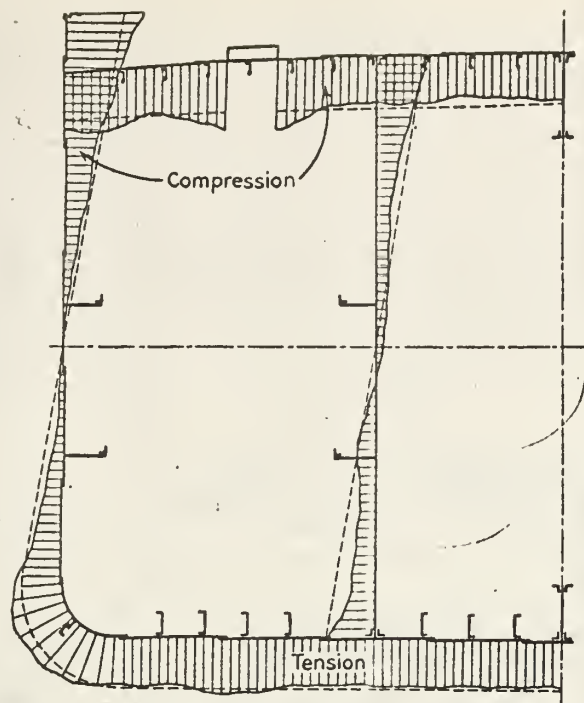


Figure 12b. - OBSERVED STRESSES UNDER TEST—TANKER

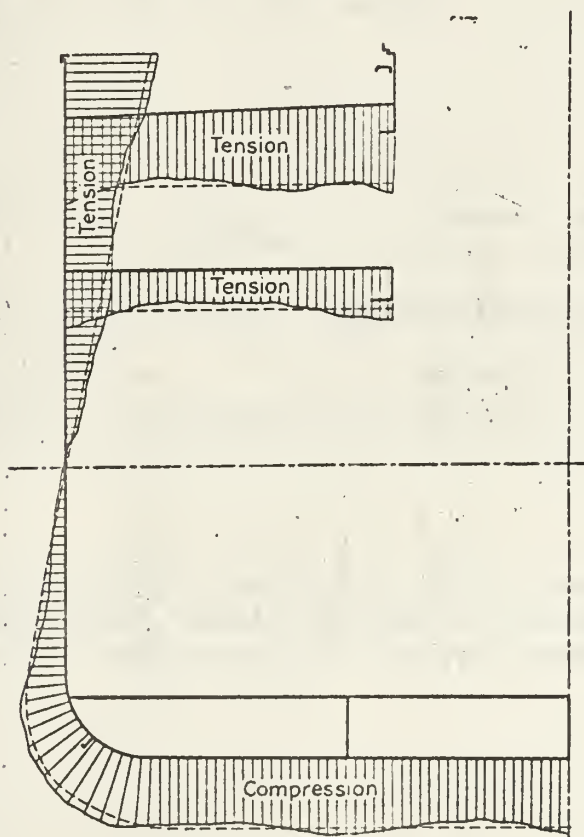


Figure 12c. Observed Stresses Under Test, Cargo Vessel.



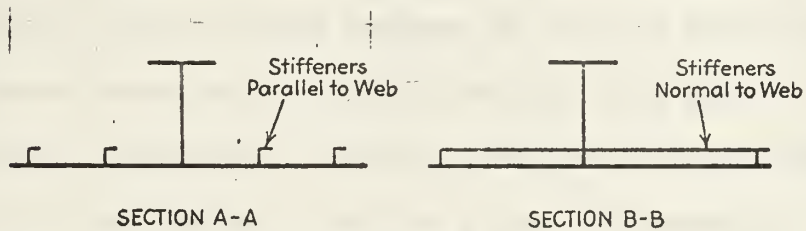
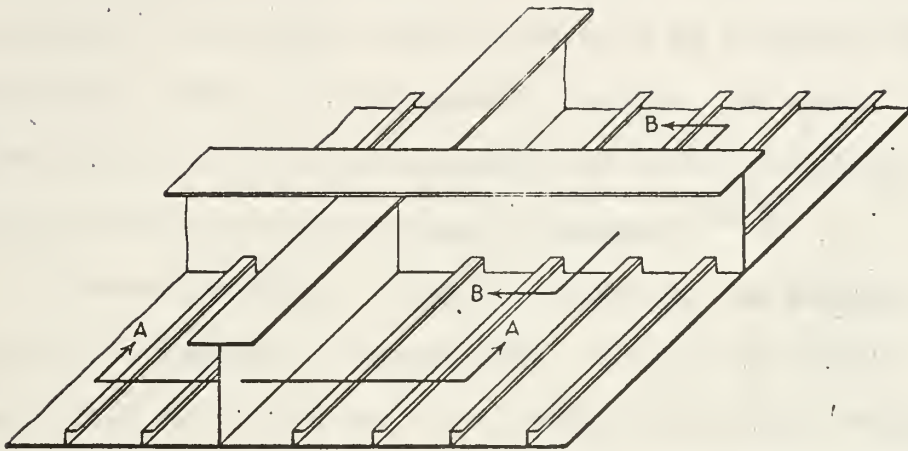


Figure 13. - ARRANGEMENT OF WEBS AND STRINGERS

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Grillages, per se, are complex structures but lend themselves well to numerical methods, wherein the fourth order differential equations describing the behaviour of an elastic girder on an elastic foundation are subjected to certain boundary conditions and the load function is presented in terms of the LaPlacian transform. The equation readily reduces to an algebraic form.<sup>\*</sup> Such exotic methods are not normally required, but when discontinuous loads and beam asymmetry is encountered these sophisticated approaches are almost mandatory.<sup>22,55</sup>

As noted previously, stress concentration and fatigue loading are important considerations. Further, no ship is constructed free of stresses. In welded construction, "locked in" stresses result from welding and can be compounded if a "stress minimizing" welding sequence is not followed. An example of a proper welding sequence is shown in Figure 14.

Stress concentration at hull openings is of great importance. Particularly when one considers that many major openings in the flanges of the box girder are essentially rectangular, such as hatches and condenser cooling water inlets.

The course syllabus contains the topic of circular holes in plates. Consider now a hole on the order of 20 by 30 feet in a plate, and the plate subjected to alternate bending (flexing) in addition to torsional flexure; the loading encountered in an operating environment - not a simple problem.

The approach to a solution of stress concentration is

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\* Recall that Macauley's method started with a fourth derivative, which was set equal to the load distribution.



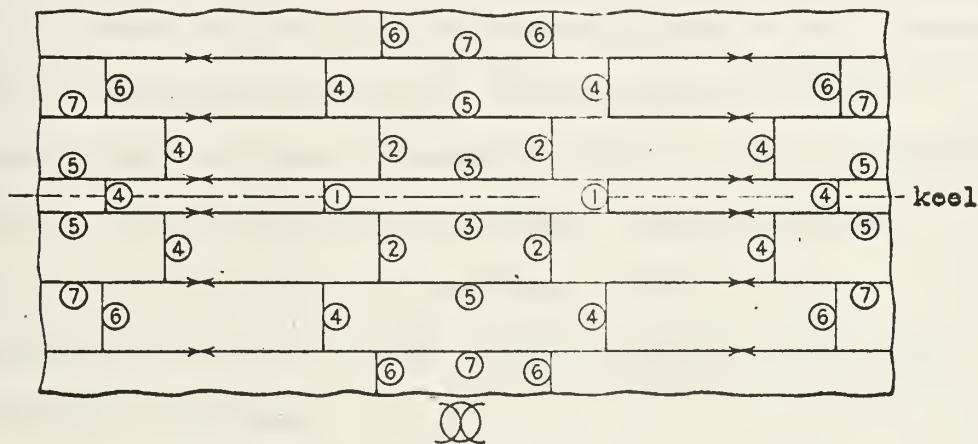


Figure 14. - WELDING SEQUENCE

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exactly as covered in ME 522.<sup>23,24, 25</sup> Allow for high stress concentration at the edge of the hatch: actual fracture may be prevented by plastic flow of the material, but when the load is alternating, fatigue failure may occur quite rapidly.

Figure 15 describes a familiar diagram. Here the results of investigation of square holes is described in terms of stress concentration factors.

One cannot arbitrarily make hatches as large as one pleases. There is indeed an optimum size. Stress analysis techniques discussed have been used to determine not only stress concentration, but the optimum arrangement of openings. References (10,26) describes a purely analytical approach, reference (27) an investigation employing brittle lacquer, reference (28) using photoelastic techniques.

Hatch corner design requires careful consideration, employing generous fillet radii in supporting gussets and the use of doublers. Figure 16 describes a typical design, Figure 17 shows a similar hatch corner in an exploded view. The designs shown are typical, but not the best.

It may seem that much ado is being made about hatches and hatch corners. There is a good reason. This particular subject led to a critical problem during World War II, as will be seen shortly. The figures are included here for pictorial representation only.

Even cutting access holes in any beam or girder deserves careful consideration. The cut must be made near or at the neutral axis to avoid the areas of highest stress, and the cuts must be generously filleted or reinforced in some manner. When the beam is used in continuous support, such as for deck





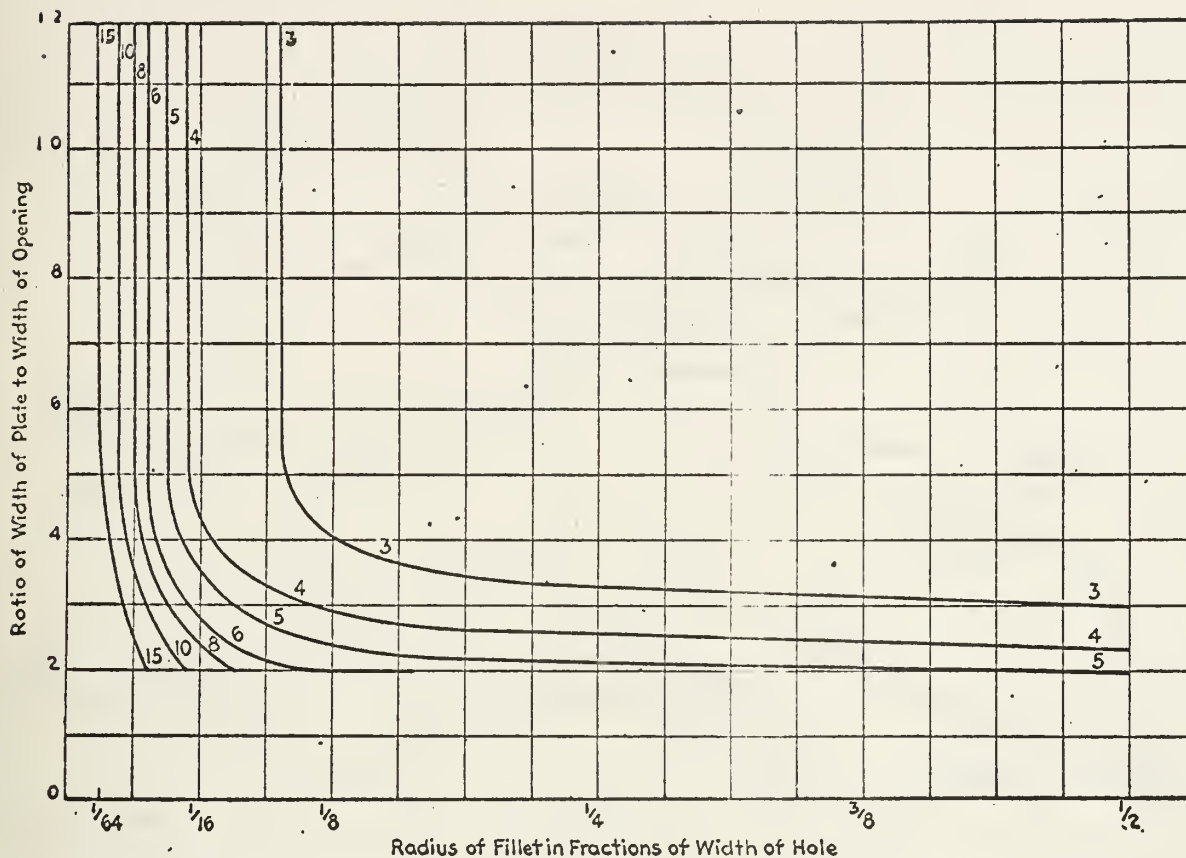


Figure 15. —EFFECT OF RADIUS OF FILLETS ON STRESSES AROUND RECTANGULAR OPENINGS IN PLATES

Values on curves are factors of normal tensile stress.

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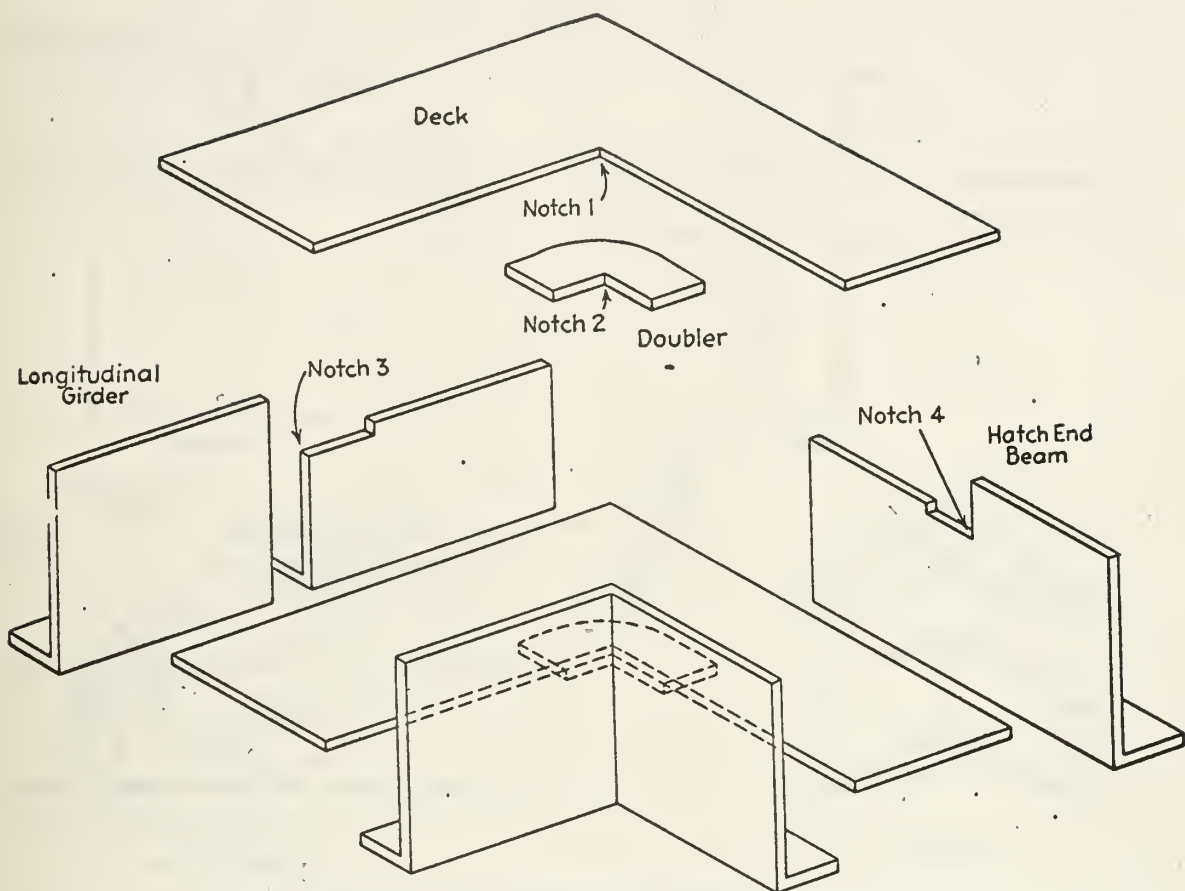


FIG. 16—LIBERTY SHIP HATCH CORNER ASSEMBLY AND EXPLODED VIEWS

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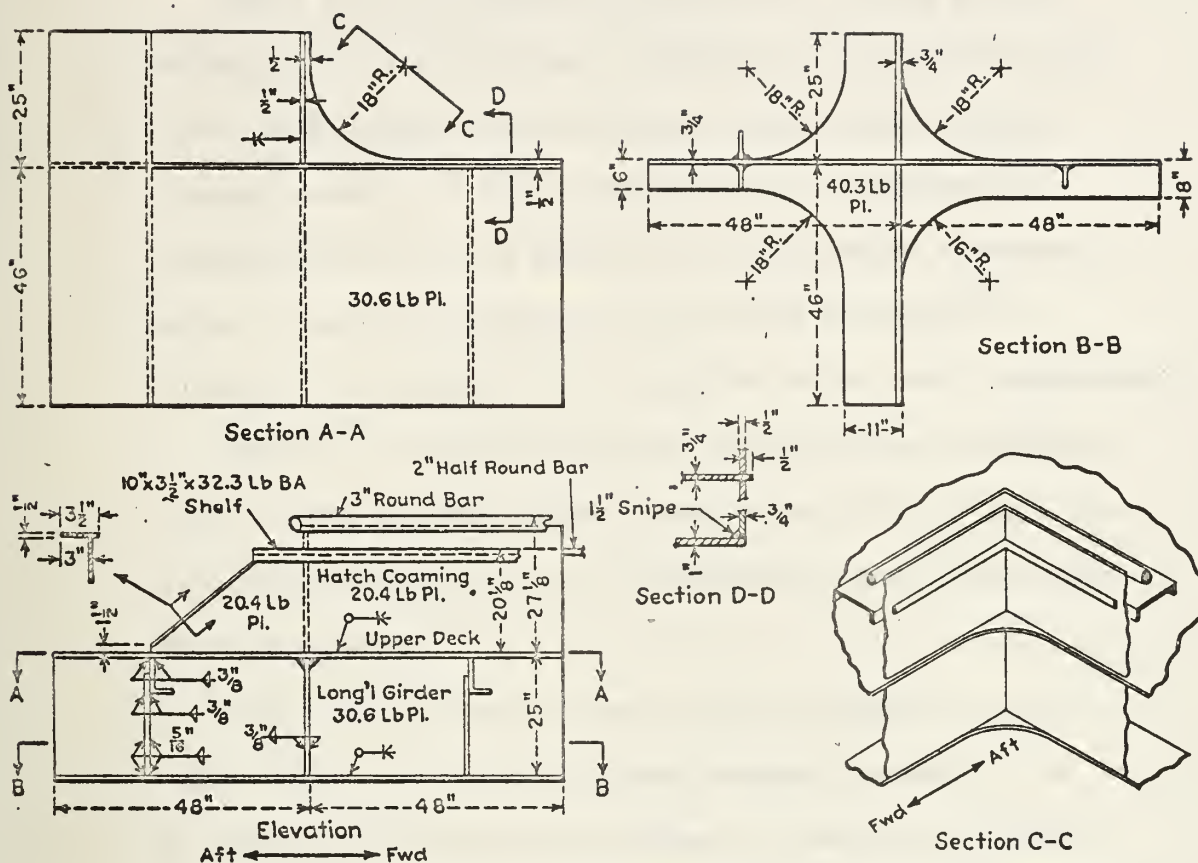


Fig. 17 Victory ship hatch corner

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plating, the neutral axis is not at the geometric centerline of the beam. Figure 18 describes the general problem in an adequate manner.

Prior to, and during the first year of World War II, welding in shipbuilding was a relatively new innovation and little used in major shipbuilding for major hull joining although several all welded ships had been constructed. Riveting was the primary method for joining major structural parts, at least of effective longitudinal material.<sup>2a</sup> Welding was in somewhat more extensive use in naval construction.

The need for mass production of ships became painfully clear in 1942 when the U-Boats were having a field day. This need dictated faster means of construction, and welding was one of the means.

There was, to say the least, some resistance to all welded ships. Too rigid, too weak, untried methods and so on. Soon, all welded ships started to flow from the yards, even yards that were not in existence a year before. This story in itself is a fascinating one.<sup>29</sup>

However, some of the all welded ships, and some of those that were mostly welded, promptly started, not only to crack, but to break in half, and not always in heavy weather. The tanker SCHENECTADY broke in half at her outfitting pier, even before she was put into service. Here, the effect of a still water bending moment, in conjunction with other factors, is seen. It was estimated that the ship broke completely in half on the order of  $1/40$  of a second.

What was the cause? Design? Note that merchant ships are not always designed from the keel up, so to speak. Various





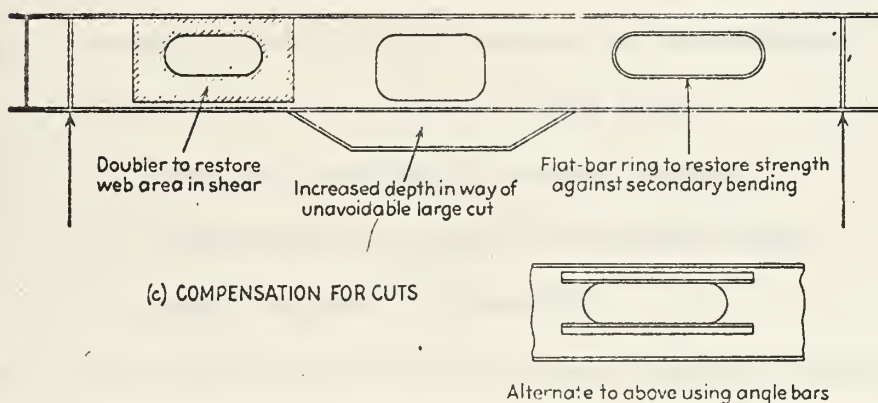
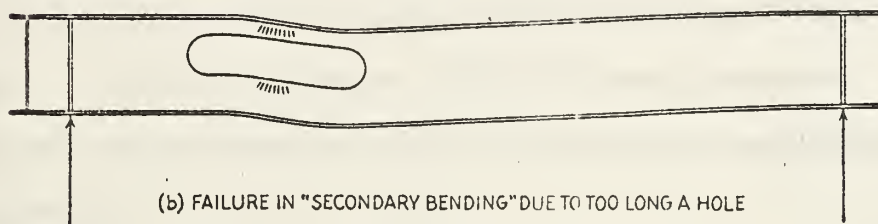
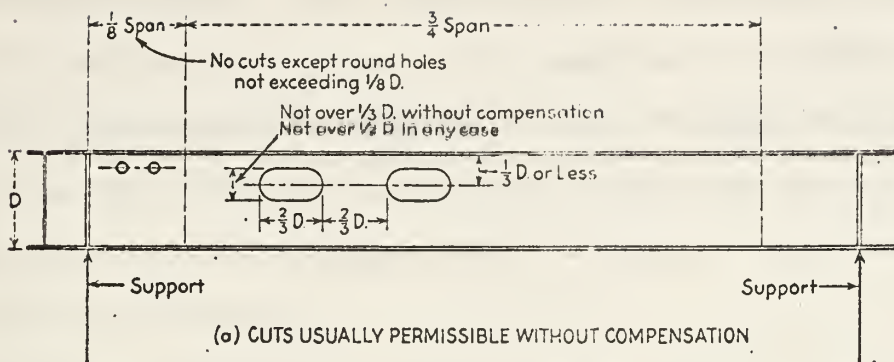


FIG. 18 CUTS IN STRENGTH STRUCTURE

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classification societies have published rules for the construction of merchant ships.<sup>6</sup> These rules are almost universally followed. If the builders follow these rules, ships may be constructed without any strength calculations at all. The rules are based on experience, with backing theory to be sure, and the service experience of such design has shown the ships so constructed to have adequate strength.

Riveted ships were not free of cracks. The S.S. LEVIATHAN was plagued with severe cracking during her entire lifetime. Riveted construction provides "built in" crack arrestors. Hull cracking was not new, but had never reached catastrophic proportions.

It was this situation that led to a full scale investigation of ship structure, materials and methods of construction.

By 3 June 1944, from a total of 2993 ships:

95 developed potentially serious cracks,

20 suffered complete loss of strength deck,

5 broke completely in two.<sup>30</sup>

A small percentage, but even one suffering complete failure is sufficient cause for concern.

The final tally was even more staggering. The complete investigation, completed in 1945, indicated that the cause of plate fracture was notch sensitivity, low temperature and temperature changes.<sup>31</sup>

The resistance to failure, under load, of any structure



is a measure of the structure's energy absorption characteristics. In this respect brittle and ductile materials are at opposite ends of the scale. Brittle material will absorb very little energy prior to failure and undergoes little, if any, plastic deformation. Ductile materials absorb considerable energy prior to failure and undergoes considerable plastic deformation. The same steel can exhibit both properties, dependent upon temperature and thickness.

A standard measure of energy absorption is the Charpy V-Notch Test. Steel, which is ductile in a tension test, may exhibit brittle features when subjected to impact loading in the notched condition. The tensile test is not sufficient in itself to determine the behaviour of steel in all conditions. The Charpy (and Izod) tests supplement the tensile test. The tests are not a direct measure of behaviour in a shock environment nor do they have any bearing on ballistic behaviour.<sup>32</sup>

In the investigation of ship failures it was found the steel used in construction was notch sensitive. Total sensitivity depends upon many factors : temperature, grain size, rate of loading, geometry, plate thickness and others. For example, plating over 1/2 inch in thickness may exhibit brittle properties but a plate of the same steel less than 1/2 inch in thickness may exhibit only ductile properties.

Temperature effects were given careful scrutiny. Figure 19 shows a typical steel energy absorption curve. Note that in effect there are two transition temperatures, an upper and a lower. For example, a proposed structure of this material







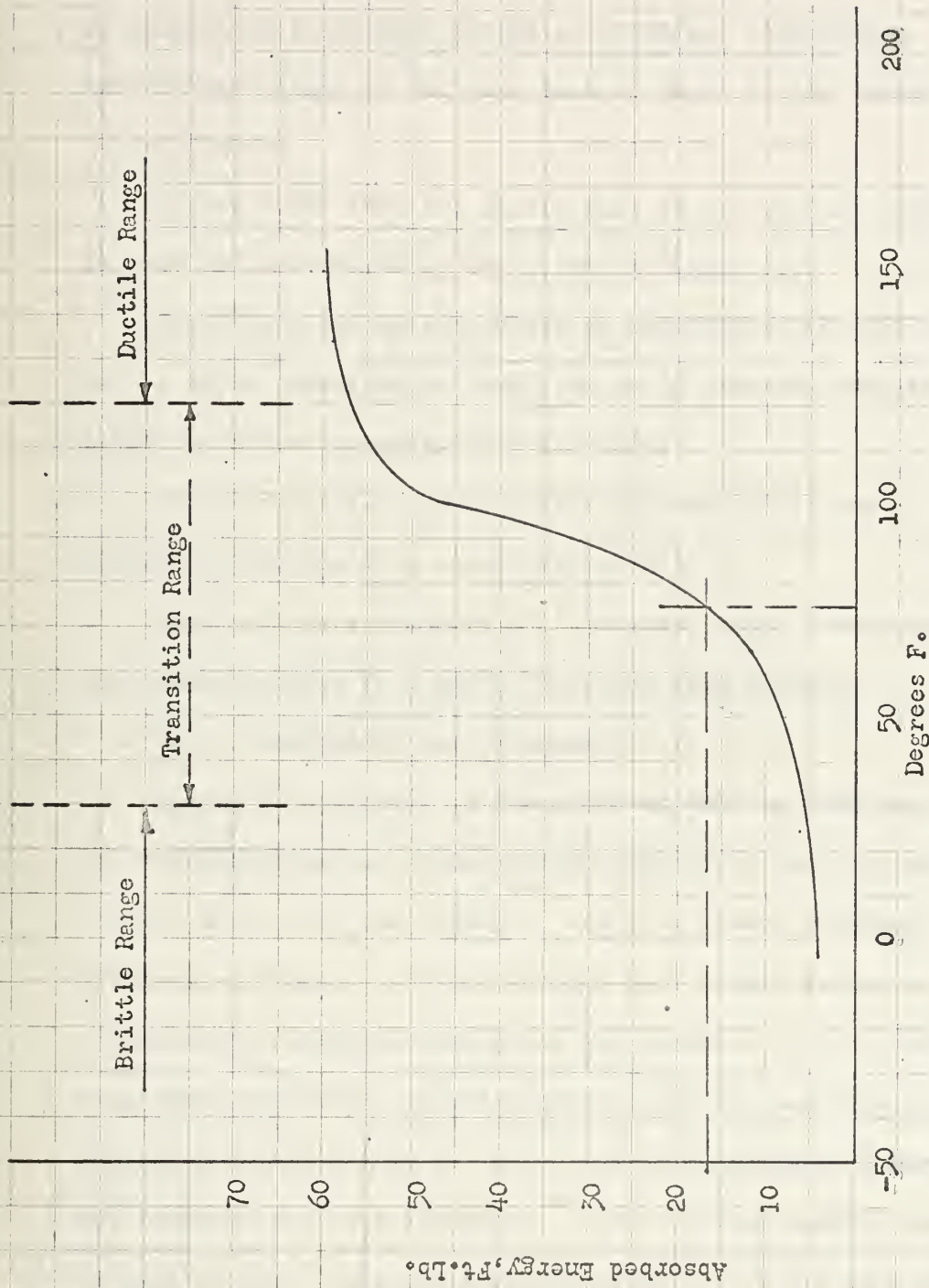


Figure 19. Typical Transition Temperature Curve for Charpy V-Notch Test



is to be used in relatively cold weather, the "middle ground" of the transition region. The curve says be careful, a decrease in temperature may lead to failure of the structure. For purposes of comparison of various steels an arbitrary "transition temperature" might be defined, such as shown by the dashed line in the figure.

It was found that the steels used in the wartime construction program is represented by the curve in Figure 19.

Grain size can be controlled in manufacture to some extent, but is quite expensive to reach the grain fineness required to shift the curve appreciably to the left.

Subsequent metallurgical investigations led to new specifications for ship steel (Figure 20).\*

Figure 21 is a comparison of various, newer steels, with the steels removed from ships that underwent failure.

The investigation may be summed:

Locked in stresses, on the order of 4000 to 8000 psi, are prevalent and may or may not be reduced by service loading. Fatigue does enter the picture, but in a lesser position. Notching, of plates, whether through poor detail design or poor workmanship, plus high transition temperatures in the normal ship operating range, provided the trigger for the ship failures. Fatigue possibly helped in some cases in progressive lowering the resistance of the structure.\*\* The bending moments were as calculated from theoretical considerations, but the beam could not resist them.<sup>30,31</sup>

Much note was made of hatch corners earlier, not without a

---

\* Alloying elements used to control grain size vice heat treatment.

\*\* Fatigue is not restricted to hull considerations, there was concern and continuing consternation over tailshaft failures during the same period and later. See reference (33).



# CURRENT ABS REQUIREMENTS FOR STEEL PLATE UP TO $1\frac{3}{8}$ IN. THICKNESS

	Class A, 0 to $\frac{1}{2}$ in., incl.	Class B, over $\frac{1}{2}$ to 1 in., incl.	Class C, over 1 in.
Carbon, max per cent.....	—	0.23	0.25
Manganese, per cent.....	—	0.60-0.90	0.60-0.90
Phosphorus, max per cent.....	0.04	0.04	0.04
Sulphur, max per cent.....	0.05	0.05	0.05
Silicon, per cent.....	—	—	0.15-0.30
Tensile strength, psi.....	58000-71000	58000-71000	58000-71000
Yield point, min, psi.....	32000	32000	32000
Elongation in 8 in., min, per cent.....	21	21	21
Elongation in 2 in., min, per cent.....	24	24	24

Notes: a Plate steels produced to the requirements of Class C shall be made with fine-grain practice.

b Where steel is made by the acid process the maximum per cent phosphorus permitted may be 0.06.

c Flat-rolled steel  $\frac{1}{8}$ -in. and under in thickness, shapes less than 1 sq in. in cross section, and bars, other than flats, less than  $\frac{1}{2}$  in. in thickness or diameter, need not be subjected to tension tests.

d For material over  $\frac{1}{4}$  in. in thickness or diameter, a deduction from the percentage of elongation in 8 in. specified in Table 1 of 0.50 per cent shall be made for each increase of  $\frac{1}{8}$  in. of the specified thickness or diameter above  $\frac{1}{4}$  in. to a minimum of 18 per cent.

e For material under  $\frac{1}{4}$  in. in thickness or diameter, a deduction from the percentage of elongation in 8 in. specified in Table 1 of 1.25 per cent shall be made for each decrease of  $\frac{1}{8}$  in. of the specified thickness or diameter below  $\frac{1}{4}$  in.

Figure 20. ABS Requirements for Steel Plate.

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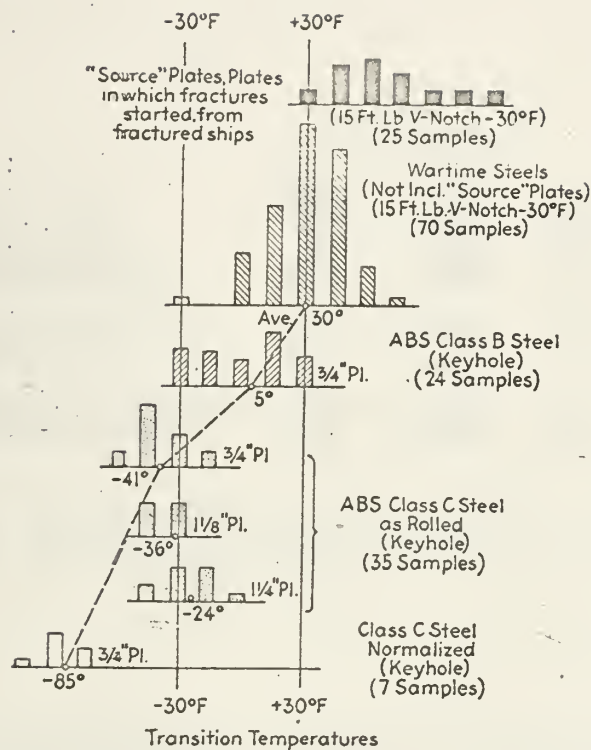


FIG. 21 SUMMARY OF SHIP STEEL CHARPY TRANSITION TEMPERATURES

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Hatch Corner Tests						
No. specimen	Design feature	Ultimate strength, psi	Energy absorption, in.-lb	Energy ratio	Ship-month service time	Fracture per ship-month
5	Basic Liberty square corner	24000	230000	1.0	22146	$0.95 \times 10^{-3}$
28	Rounded gusset-plate reinforcement	31400	974000	4.3	17115	$0.22 \times 10^{-3}$
30	Rounded doubler slotted through coaming	34200	3938000	17.0	7722	
31	Oval doubler square corner-British detail	30400	1990000	8.7		
34	ABS Design (Victory ship detail)	33200	5800000	25.0	32340	$0.0031 \times 10^{-3}$
35	Kennedy design-double curvature	54100	6786000	29.5		

Figure 22. Energy Absorption Characteristics of Various Hatch Corner Designs.

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reason. The majority of cracks started at hatch corners, particularly in the Liberty ships, which were the first product of the expanded building program. Later designs, such as in the Victory ships, were much better. Figure 22 shows comparative energy absorption characteristics of various designs.<sup>21</sup>

To this point, discussion has centered on merchant ship design and construction. Before proceeding, some variances in warship design must be noted.<sup>3,34</sup>

Comparable to merchant practice, hydrostatic loads are determined. Next, however, the ballistic requirements are determined and then the midships section is developed. Where possible, ballistic plating is worked into the structure. In general all of the hull material in the major portion of the hull is of higher yield than used in merchant ships. The ballistic material worked into the hull structure is STS and HY series steels (historically Class B armor). Class A armor (side armor, little used in current construction) is not meant to contribute to structural strength, it is merely placed on a "shelf" and bolted to the hull and backed by concrete and bitumastic compound.

Structural design requirements of warships today are quite different than that of World War II. Obviously, a warship must be able to receive some damage and survive. Damage here is meant to imply some loss of effective longitudinal material, or, at least, some disruption in longitudinal continuity of effective material. World War II designs were predicated on the basis of absorbing damage from contact hits (explosions). Capital ships also were designed to withstand multiple torpedo



hits. This consideration is reflected in the side protection systems and in the double and triple bottom configurations.

Prior to and during World War II the U.S. Navy had been (and still is) investigating the ability of structures of all types and configurations and construction methods, including welding, to absorb explosive energy. Quite often the tests involved the use of large scale models.<sup>35</sup> Many tests had been carried out prior to the merchant ship cracking problem described earlier. The results of these tests did aid in the cracking investigation.

In any case, the problem of ballistic protection is in fact design for energy absorption. It could be termed plastic-failure design. Ballistic protection does not in any sense mean total defeat in penetration of the projectile. Many various schemes have been proposed, Figure 23 notes two. Figure 24 displays the results of a typical contact explosion. The damage is extensive, but local. If the design is successful, and the contact charge is equal to or less than the design charge, the outer and perhaps the two outer hulls will be breached, but the innermost, although perhaps undergoing radical plastic deformation, is intact and watertight, but less effective in longitudinal strength.

Specific details are to be found only in the classified War Damage Reports, but certain declassified summaries have been published.<sup>36</sup>

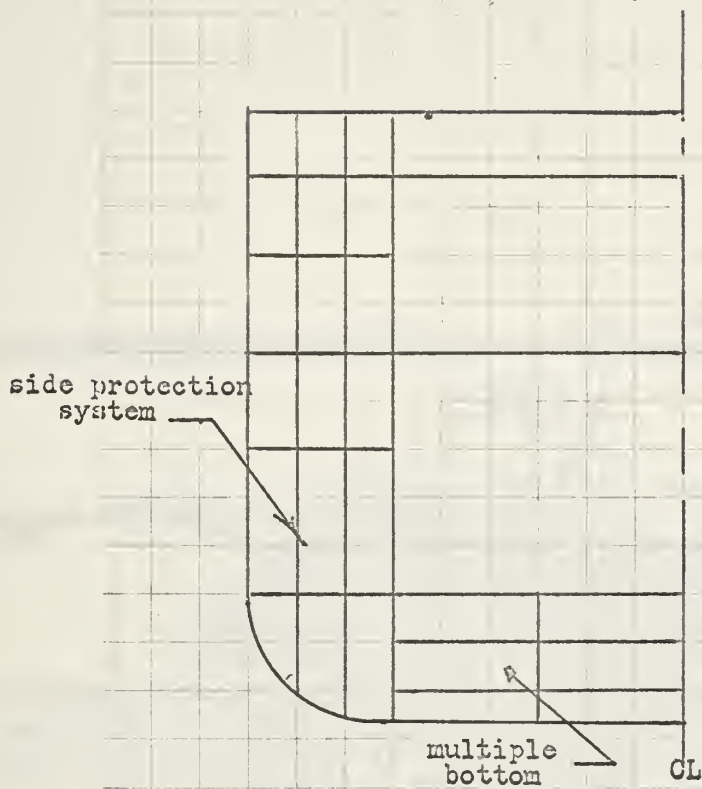
Non-contact mines and torpedoes encountered during the war produced somewhat similar damage, but the hull loading was markedly different. The hull can be loaded in two ways, either by initial shock wave or by succeeding bubble pulses. Figure 25

shows what happened to the hull of a certain ship when it was

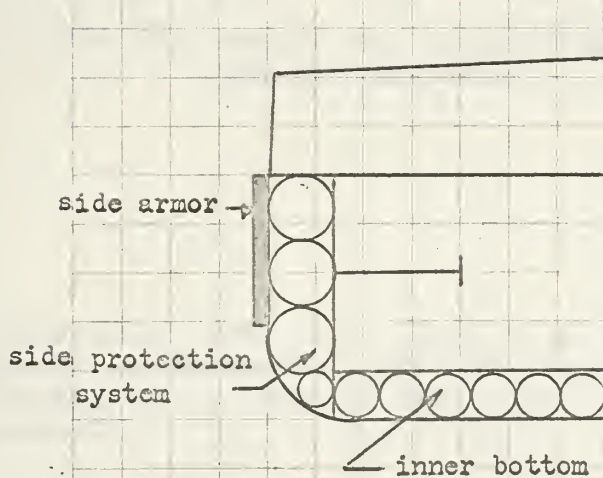








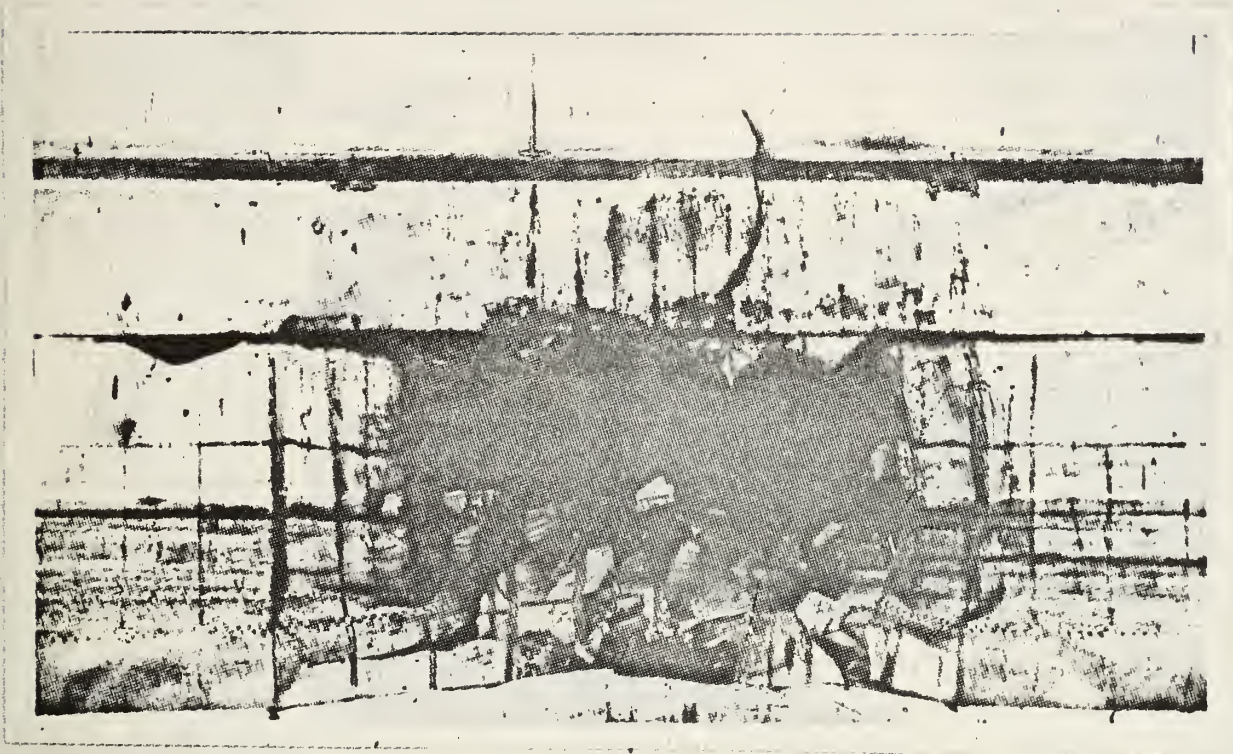
Typical Aircraft Carrier



Proposed Use of Tubular Members

Figure 23. Side Protection Schemes for Explosive Energy Absorption.





**Figure 24. Results of a Contact Explosion Against the Side of a Cruiser.**

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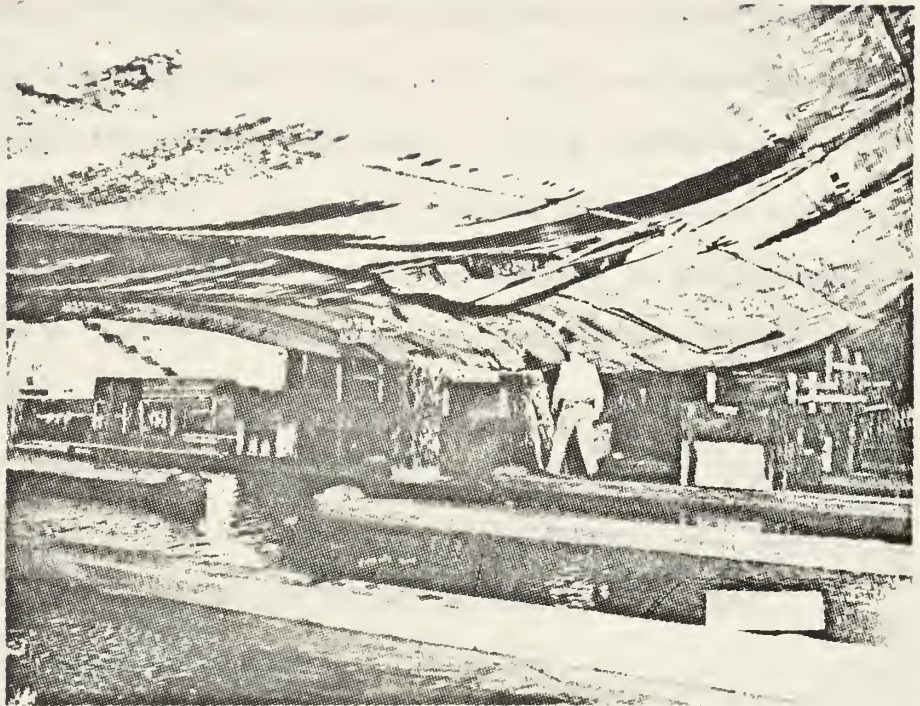


Figure 25. Bottom Damage to a Cruiser Through Bubble Pulse Loading.

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shows what happened to the bottom of a cruiser subjected to this type of dual loading, produced by a small charge weight. Shock wave loading, complex as it is, is simpler than bubble pulse loading, which only recently has been reasonably explained.<sup>37</sup>

Obviously, the structural requirements must be in excess of that based on a bending moment alone. The usual procedure is to design for a specified amount of damage, leaving sufficient hull strength to survive. Also, the residual calculations must take into account the possible stress aggravation in the damaged condition due to probable flooding, which changes the bending moment.

Nuclear weapons have changed some basic design criteria. The necessity for a change was proved beyond doubt in the two nuclear shots of OPERATION CROSSROADS, particularly by Shot BAKER, an underwater detonation.

The loading here is primarily by plane shock wave of high intensity, whether the detonation is in air or underwater, as contrasted to the spherical propagation of charges of conventional size and composition. A comparison of pressure-time histories is shown in Figure 26. In both cases the pressure decay can be approximated by the exponential expression:  $P = P_{\max} e^{-t/\theta}$  where  $P$  is pressure,  $t$  is time and  $\theta$  is the time constant (unique for each explosive).

What the structure feels as a result of this type of loading is described in Figure 27. The energy transfer to, and absorption by, the hull is extremely complex.<sup>39,40</sup> Not all of the energy contained in the pressure-time envelope is transmitted to the hull. Upon arrival of the shock wave, the plating starts to move rapidly, so rapidly that cavitation





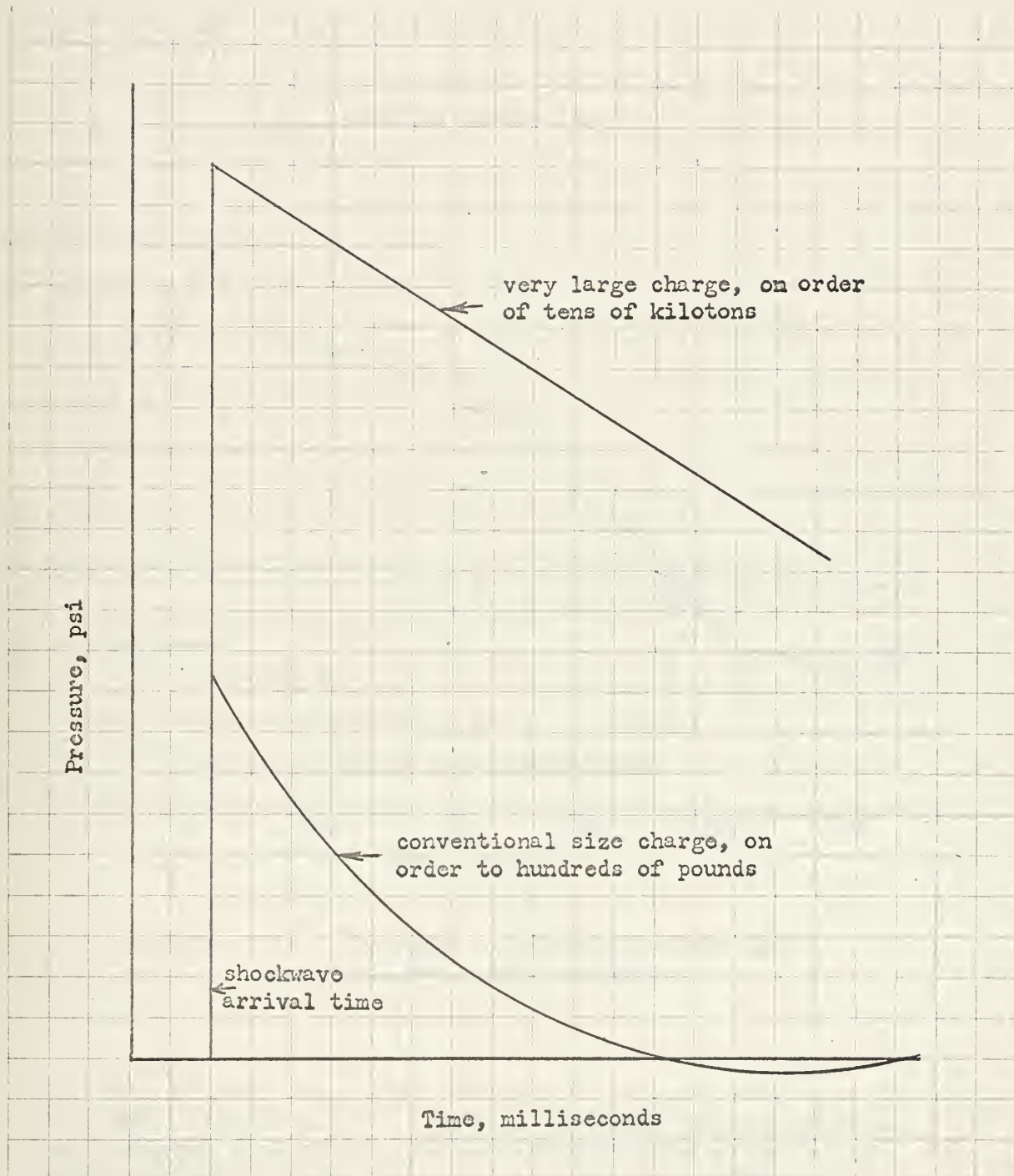


Figure 26. Comparative Pressure-Time Histories.



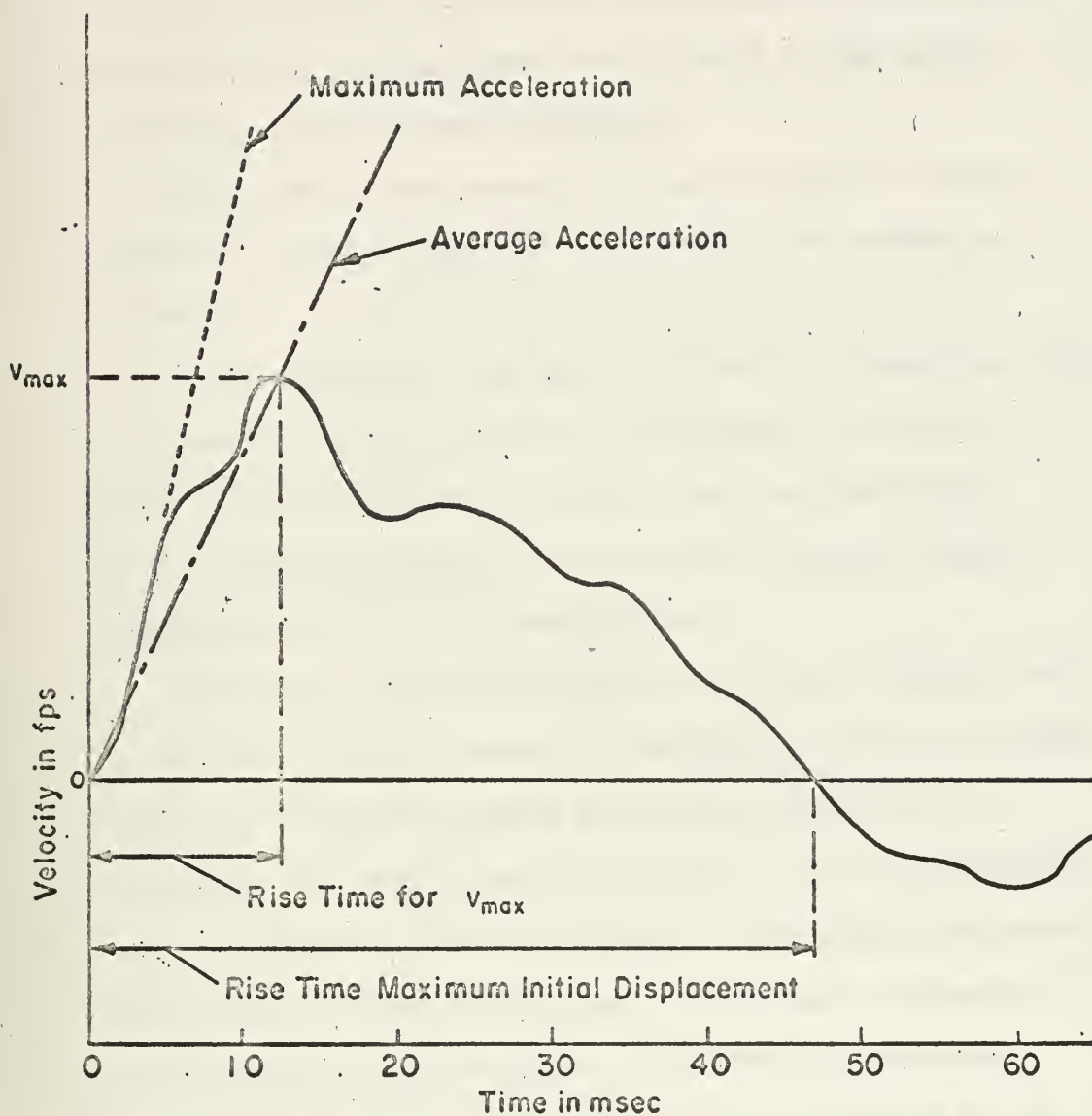


Figure 27. Example of Shock Motion and Terminology (Mild Shock)

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occurs at the wetted surface of the plate. The cavitation acts as a barrier to further energy transfer.\* The plate will stop, the cavitation collapses and the pressure builds again. The plate will stop again and start to move in the initial direction, cavitation may occur again.

Also, the incident energy is a function of the "attack geometry" and may produce only side damage, as is shown in Figure 29.

Scale models have been used extensively in these investigations. Full scale testing has been far less employed. Model and full scale tests have provided a wealth of data and reasonably good correlation. Figure 30 shows typical destroyer damage resulting from plane shock wave attack.

The energy that is transmitted to the hull manifests itself in many ways. Elastic waves run throughout the ship. The hull acts as an attenuating medium and certain frequencies are associated with various parts of the ship. The ship definitely does not respond the same throughout. The elastic waves travel along until a wetted plate is met, some energy is transmitted to the water and some reflected. The process repeats until naturally damped out. The phenomenon not only allows the use of sophisticated mathematical treatments, but substantiates the theory of shock wave-hull interaction.<sup>41,43,46</sup>

Some of the energy will go into plastic deformation, both in final set and traveling plastic waves. Some energy will go

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\* A reasonably good picture of the phenomena is to be found in "Underwater Explosions", R. H. Cole, Princeton University Press, Princeton, New Jersey, 1948, Plate XII, facing page 406. Consider this to be representative of the missing Figure 28.







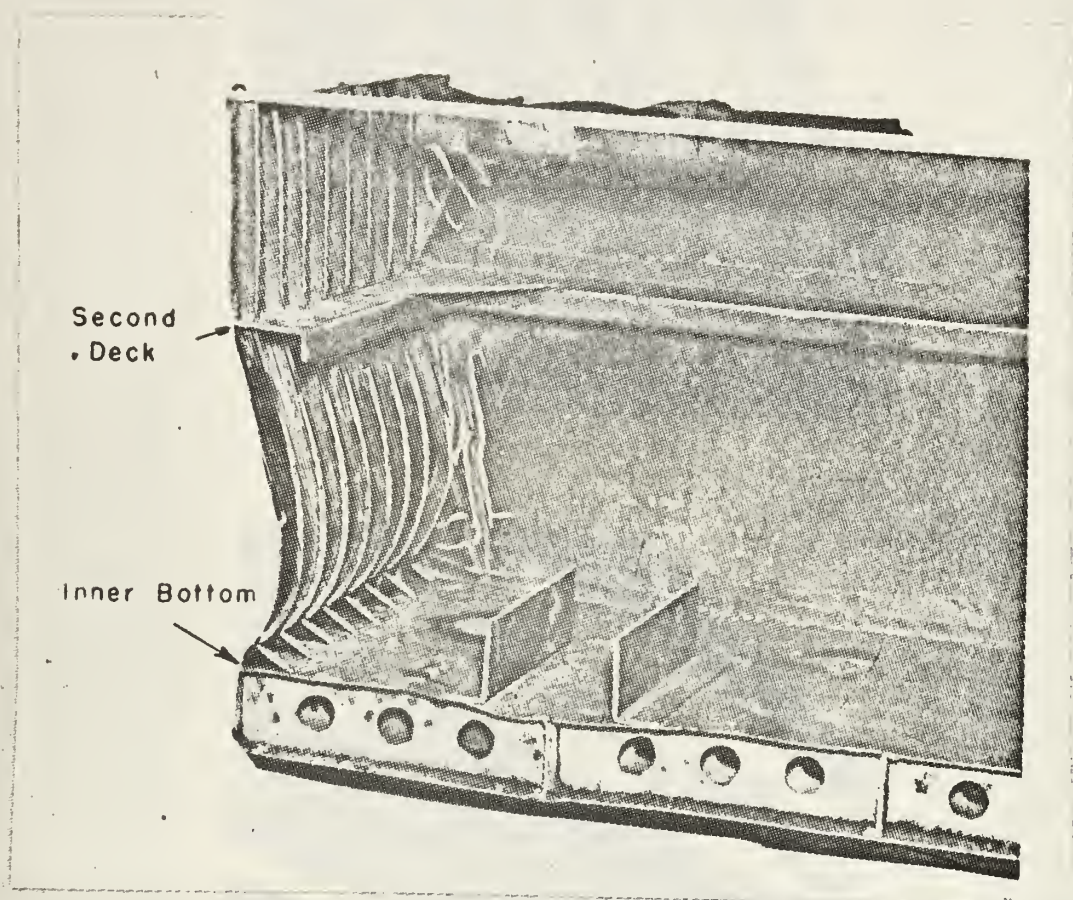


Figure 29. Results of Non-Contact Explosion, Side Attack,  
1/35 Scale Model.

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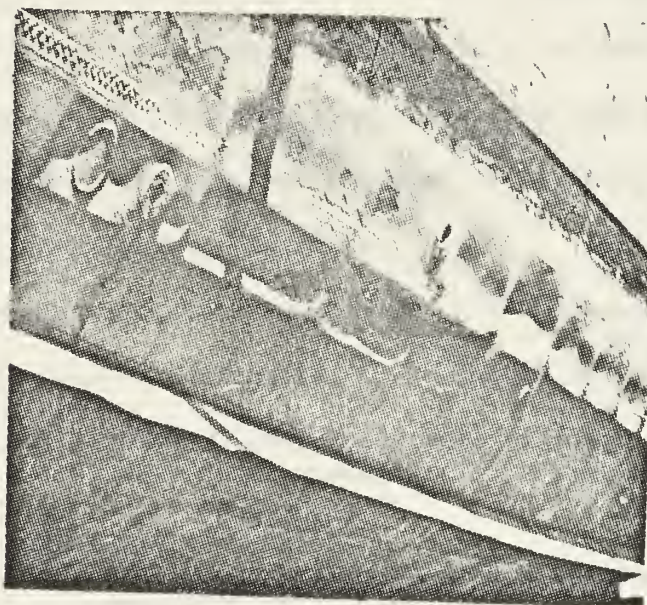
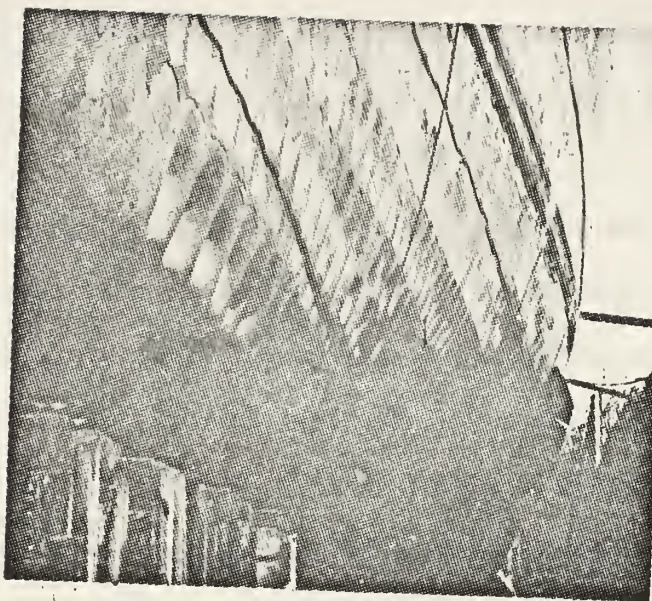


Figure 30. Side Shell Dishing in a Destroyer.

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into heaving the ship bodily. Figure 31 describes a typical shock environment.

Not only hull structure is damaged, equipment (Figure 32), main machinery supports (Figure 33) and even the crew (Figure 33a) are subject to shockwave attack.<sup>43,44,45,46</sup>

If the loading is high enough, girder failure can be achieved. Failure is rapid and dramatic. The shockwave can excite the natural vertical modes, and if the bubble pulses (a function of charge size and depth of detonation)<sup>37</sup> are in phase, hull girder failure can quickly ensue. Figure 34 shows a plastic hinge developing shortly after arrival of the pressure pulse. Figure 35 is a close up of the final set in the hull, the hinge that developed. Figure 36 shows plastic deformation to rupture of a bulkhead caused by movement of a longitudinal. Figure 37 shows local plastic failure caused by bubble pulse loading. Figure 38 shows an overall view of the ship indicating the magnitude of the girder failure.

As an aside, the natural frequency of a ship can easily be estimated by Schlick's formula:<sup>42</sup>

$$N = C(I/DL^3)^{0.5}$$

N = First mode frequency, vertical, cpm  
I = Midship section moment of inertia, ft<sup>2</sup>in<sup>2</sup>  
D = Displacement, tons  
L = Length of ship, ft  
C = Constant, dependent upon type of ship  
may vary from 1.3 to 1.5 x 10<sup>5</sup>

For almost a total side attack, transverse strength is of some importance. Little had been done concerning transverse strength analysis prior to the necessity to look at this aspect brought about by plane shock wave loading.<sup>47</sup>





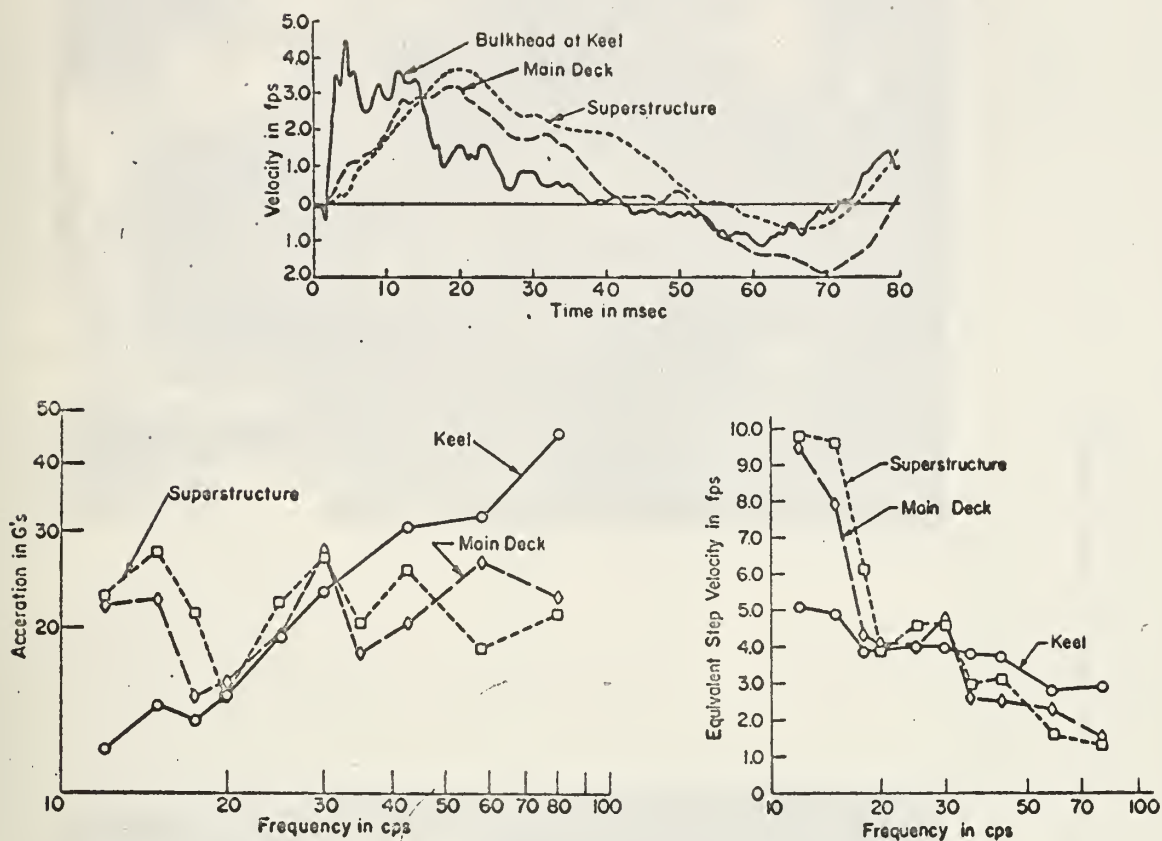


Figure 31 Variation of Shock Environment Throughout a Destroyer (For Moderate Shock)

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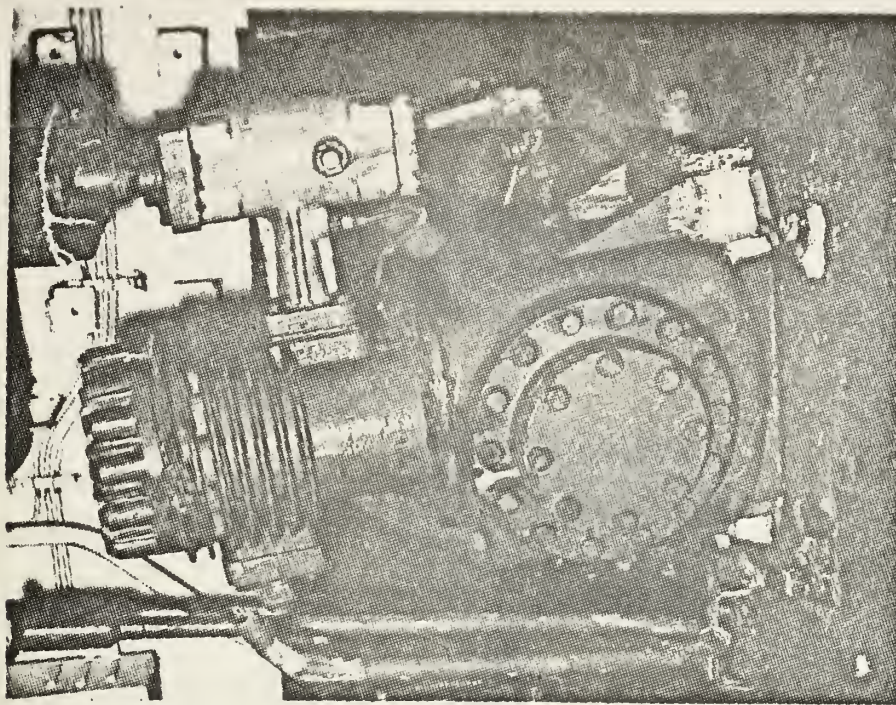
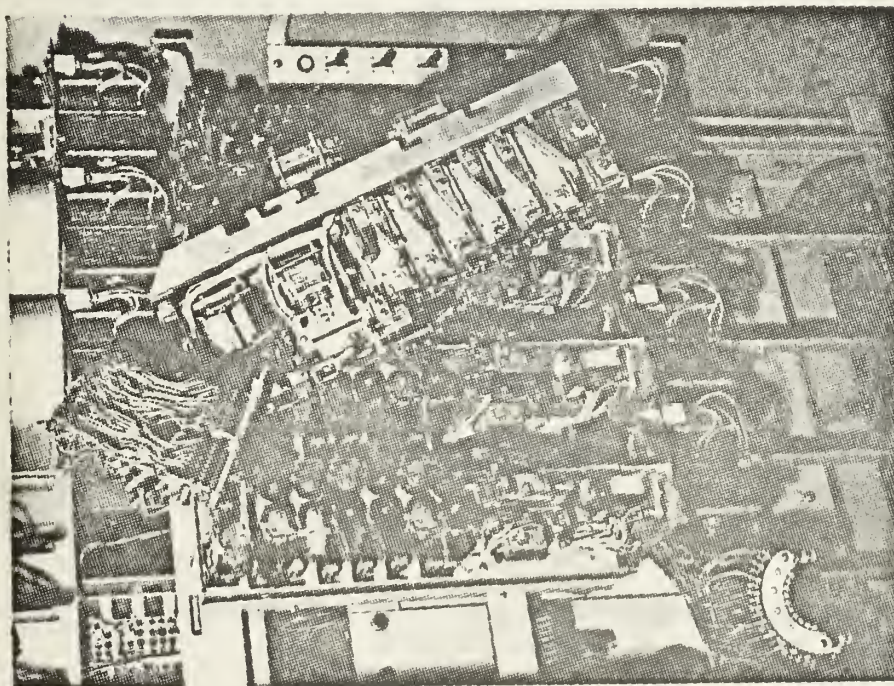


Figure 32. Equipment Shock Damage

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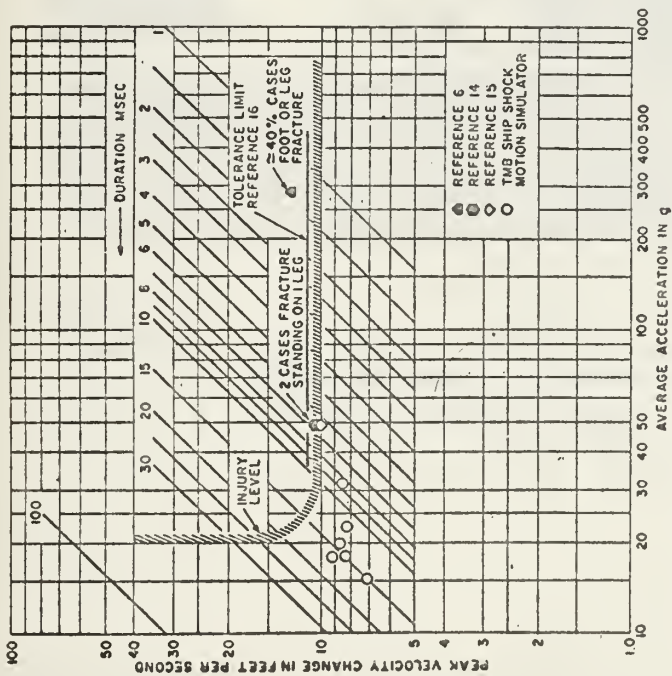




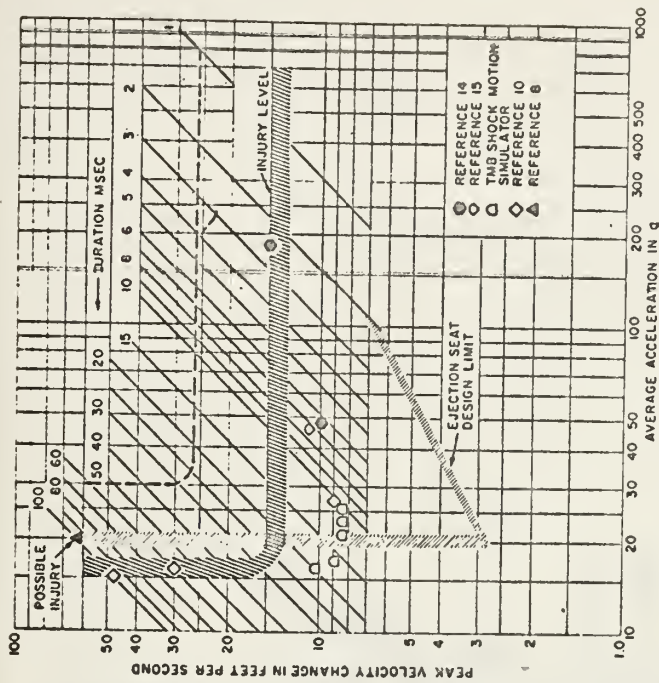
**Figure 33. Buckling of Destroyer Turbine Flexure Plate.**

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Tolerance of stiff-legged standing men to shock motion of short duration



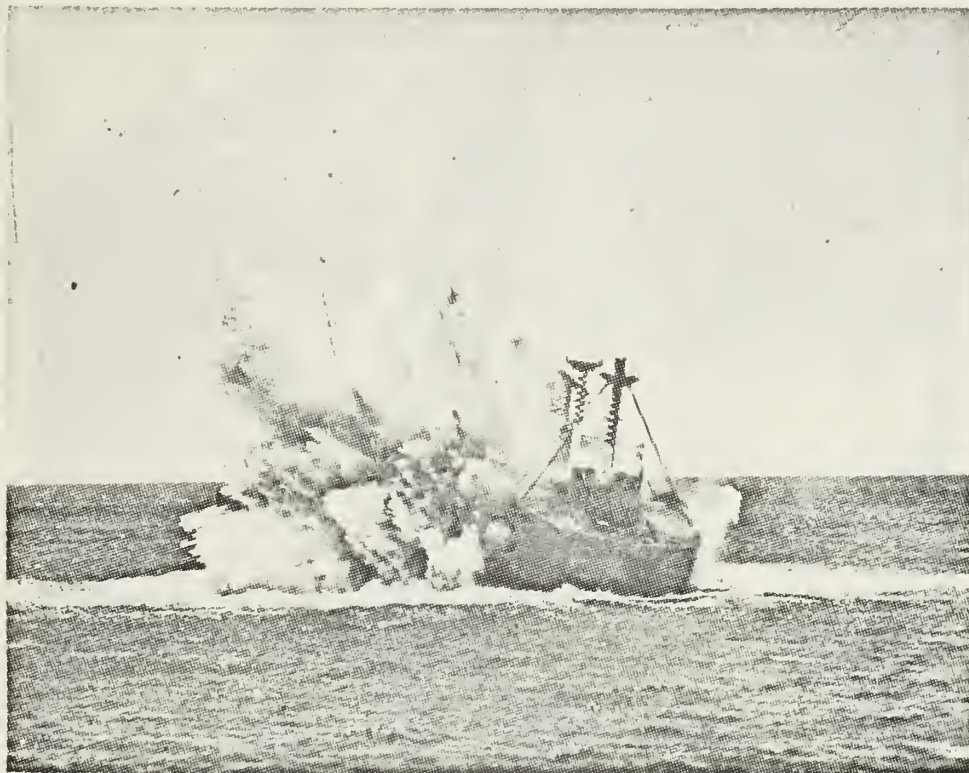
Tolerance of seated men to shock motion of short duration

Figure 33a. Human Tolerance to Shock Motion.

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**Figure 34. Start of Hull Girder Failure Resulting from Shock Loading in a Non-Contact Explosion. Plastic Hinge Developing.**





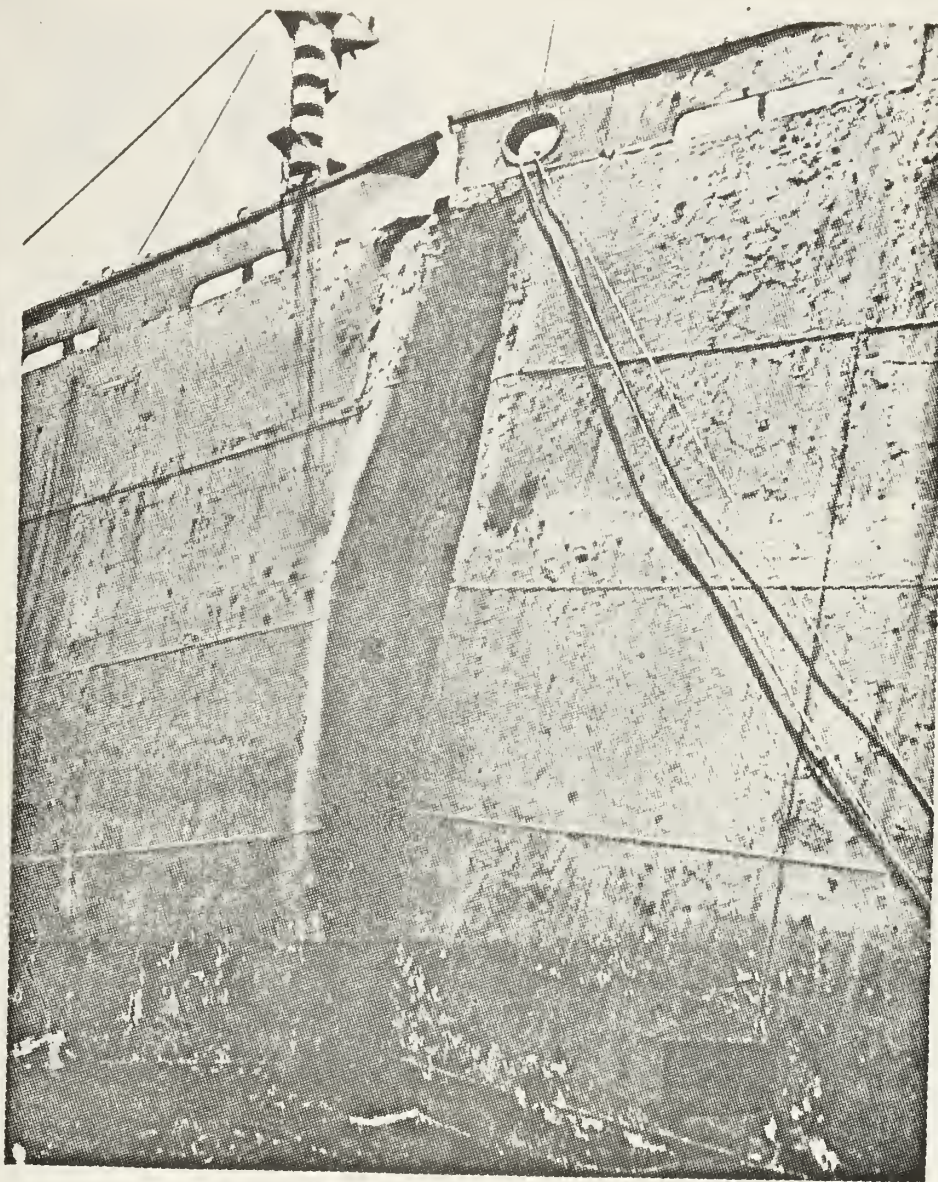


Figure 35. Hull Girder Failure.

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Figure 36. Bulkhead Plastic Failure.

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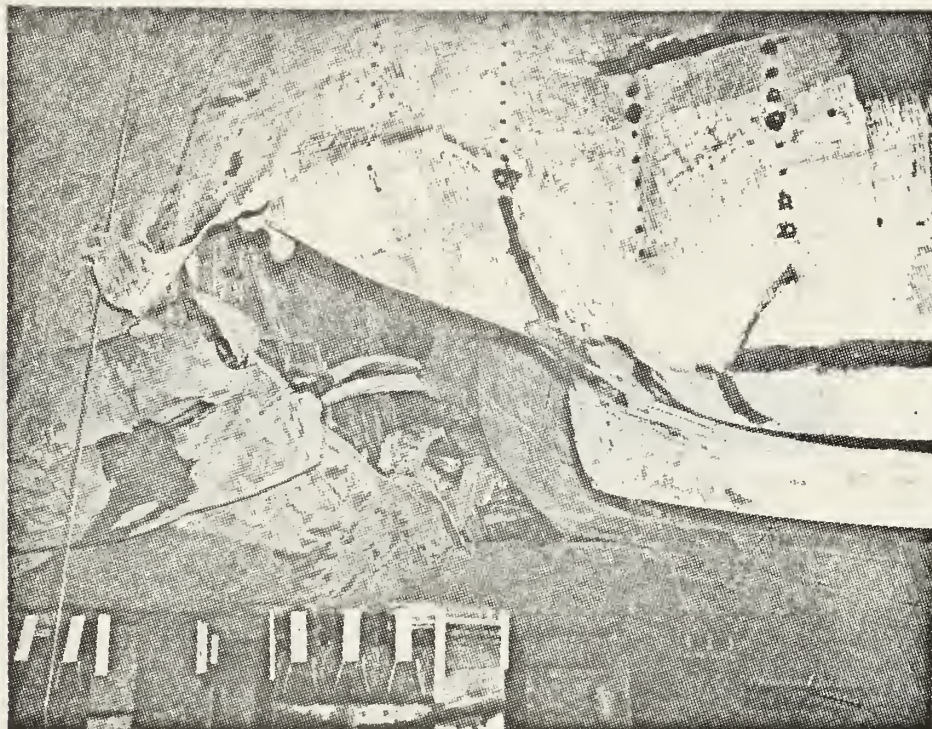


Figure 37. Plastic Failure of Ship Bottom,  
Bubble Pulse Loading.

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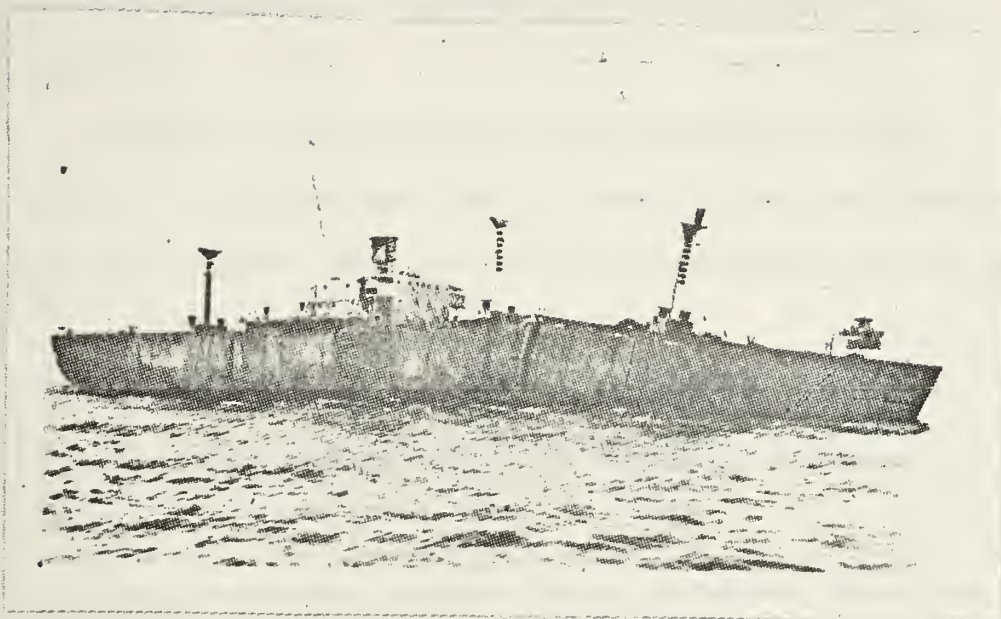


Figure 38. Extent of Hull Girder Failure.

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Blast and shock loading from new weapons can be considered to load the whole structure, there is essentially no time variation in initial application of the load over the entire structure. To design for reasonably close near-hit survival from only elastic considerations is out of the question. Plastic design approaches must be used. Such approaches are being encouraged for merchant practice also.<sup>48,49,50</sup> Why design elastically for a maximum load that may never be encountered?

Methods for investigating plastic behavior under high transient loading follows elastic methods in that small sections are first examined, then more complex structures and so on. It is a building block type of progress.

The easiest plate geometry, in unstiffened plates, to deal with is the circular plate, or diaphragm, because of symmetry.

Energy absorption characteristics of various steels can be compared. Figure 39 draws a qualitative comparison between mild steel and various naval ship steels based on tension tests.

Progressive plastic deformation of a circular plate is described in Figure 40. Both elastic and plastic waves move through the plate. Compare the final plastic profile with that shown in Figure 30.

Referring to Figure 39, consider the area under the curve:  $\sigma \frac{1bf}{in^2} \times \epsilon \frac{in}{in} = \sigma \epsilon \frac{1bf}{in^3}$ , or energy of deformation per unit volume of material.

Note that during deformation the total volume of the plate remains the same, but the surface area is much greater (the





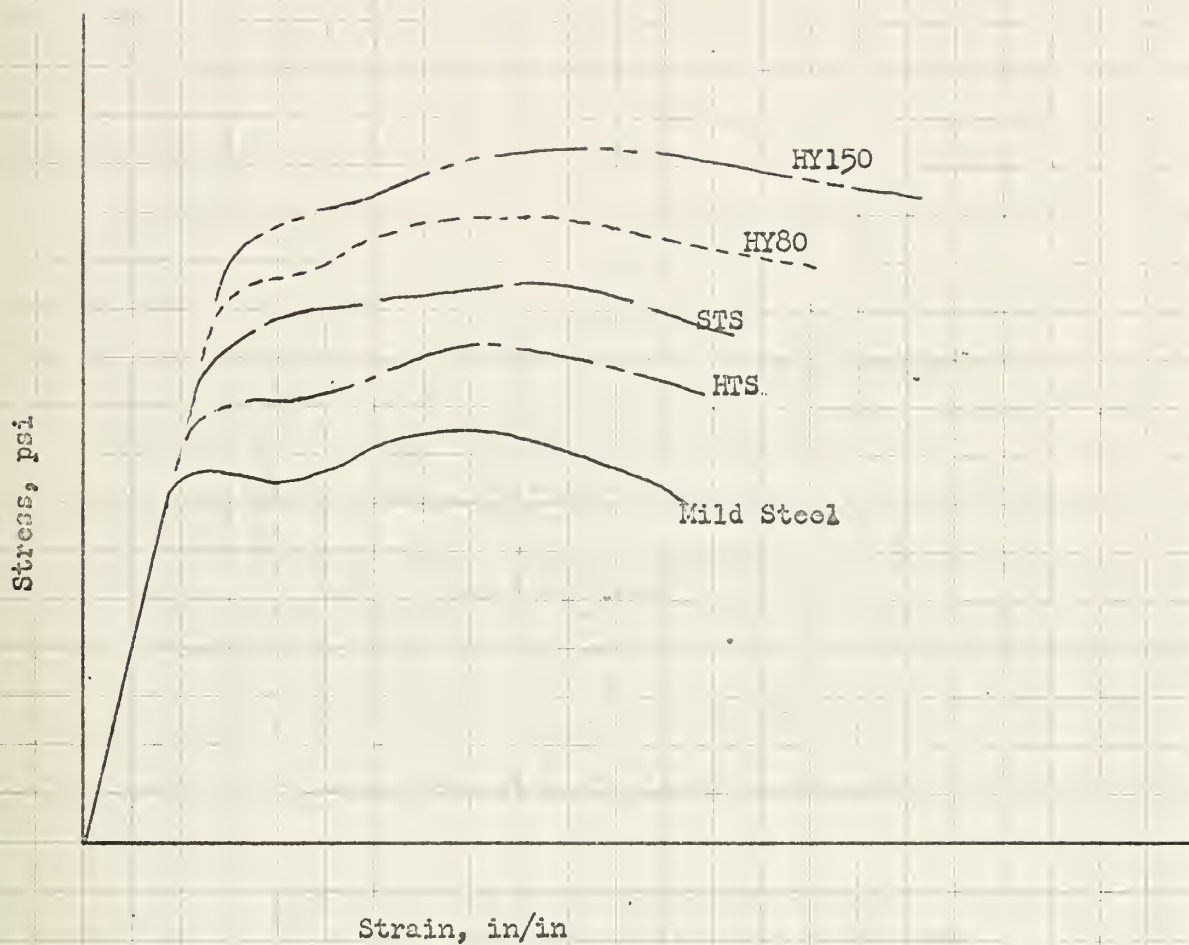


Figure 39. Generalized Comparison of Various Ship Steels.



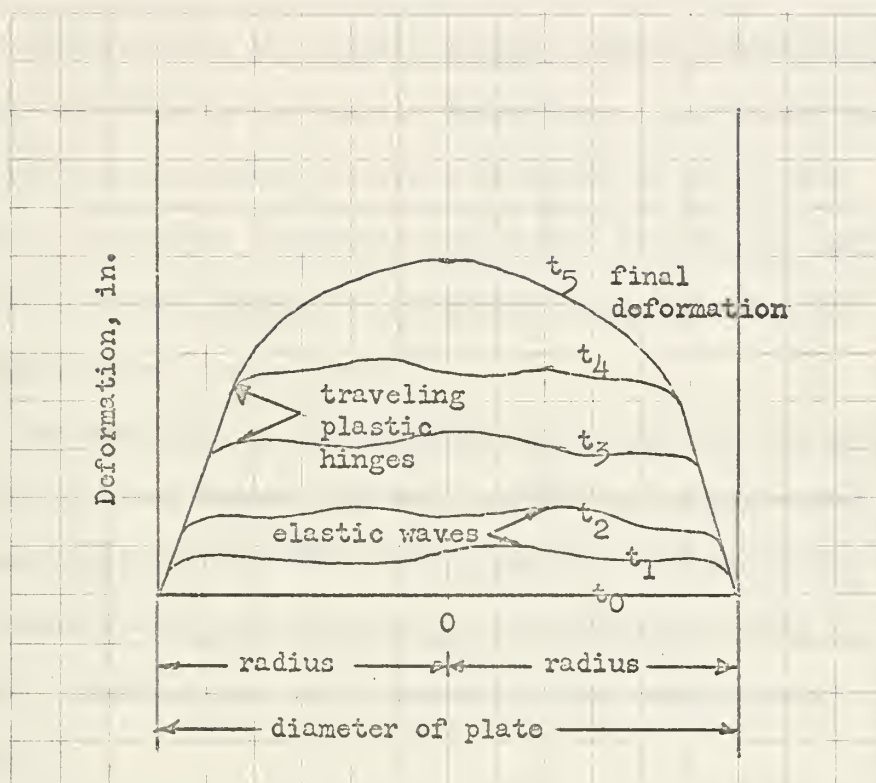


Figure 40. Schematic Representation of Progressive Plastic Deformation of Circular Plate Subjected to Underwater Explosion.



final shape is actually parabolic).<sup>52</sup> The plate therefore undergoes some thinning (non-uniform and thinner towards the center) as shown in a typical thickness strain variation in Figure 41. By suitable radial and transverse measurements a thickness strain can be determined. These strain measurements, when compared with the tensile diagram, permit calculation of energy absorbed in the plastic deformation. The Charpy tests are invalid here since the plate initially is not in the notched condition. Balancing this energy calculation against incident energy, quantitative estimates of percent incident energy absorbed can be made.<sup>52</sup>

The same type of investigation can be carried out for plates of other shapes, but edge conditions introduce some severe complications.<sup>53,54</sup> Grillages have been subjected to explosive loading and the theory developed can be used in scaled investigations with reasonably good prediction of damage.

The object of such work is to determine the strength of the ship when subjected to such a loading environment and to establish design criteria. The ideal case would be to reach a one horse shay design where structural strength ranges <sup>Are</sup> ~~is~~ comparable to that of equipment and personnel.





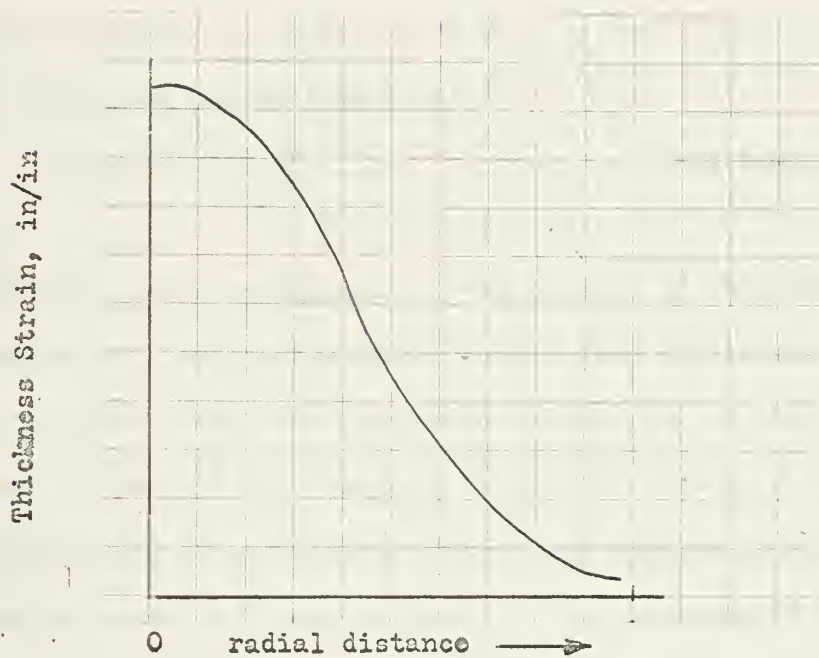


Figure 41. Thickness Strain Distribution.



## SOME FINAL QUOTES OF NOTE

"Take it all in all, a ship of the line is the most honorable thing that man, as a gregarious animal, has ever produced. Into that he has put as much of his human patience, common sense, forethought, experimental philosophy, self-control, habits of order and obedience, thoroughly wrought handiwork, defiance of brute elements, careless courage, careful patriotism, and calm expectation of the judgement of God, as can be put into a space 300 feet long and 40 feet broad."

"The Shipbuilder"

John Ruskin (1819-1900)

"Another lesson brought out by the history of ship design is the loss of ship efficiency which results from unbalance in design. In the past there have been numerous examples of ships which have suffered in this respect, frequently because of the interference of influential persons not skilled in ship design, who persisted in "riding a hobby". The capsizing of the British warship CAPTAIN with the loss of all hands is one example of this. Government owned ships have been more subject to this drawback but commercial ships have not been entirely free of it."

"The Theory and Technique of Ship Design" George C. Manning



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### Abbreviations used

SNAME - Society of Naval Architects and Marine Engineers, New York  
ASME - The American Society of Mechanical Engineers, New York  
JSR - Journal of Ship Research (SNAME Publication)  
DTMB - USN David Taylor Model Basin, Washington, D.C.  
ASNE - American Society of Naval Engineers, Washington, D.C.

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